Patents with Simultaneous Innovations: The Patentability Requirements and the Direction of Innovation*

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Abstract

We model a three-stage duopolistic game, in which firms first simultaneously choose the technological direction of their innovation, then invest in the chosen direction, and finally compete. Investments can be in overlapping or nonoverlapping technological territories, and their outcomes are uncertain. We show that compared to a regime where negligible innovations are patentable, strengthening the requirements for patentability can increase market efficiency. Importantly, we also show that the requirement level may affect the direction of firms' research and development trajectories. While in a mild patent regime firms tend to invest in overlapping technologies, stricter requirements may induce firms to operate in different technological areas, which increases social welfare and consumer surplus. We illustrate our general theory using two stylized Cournot competition models with process and product innovation.

Keywords: patents, R&D, patent requirements, direction of innovation
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1 Introduction

Patents give an innovative firm an exclusive right over the use of its innovation for a certain period of time. As a compensation for the costs and risks incurred in conducting R&D activities, exclusive rights provide the firm with incentives to innovate. Much of the literature in the economics of innovation has focused on analyzing how patent policy impacts the amount of firms' research and development (R&D) investment. Instead, quite surprisingly, the relationship between patents and the direction of innovations has attracted the attention of scholars and policy makers only recently, despite it is well known since Petra Moser's seminal study on 19^{th} century inventions that "patents help to determine the direction of technical change" (Moser, 2005).

In order to be patentable, an innovation must be new, useful, and nonobvious, which is, it must generate a sufficient amount of utility and it must be adequately distant from the state of art (Barton, 2003). Setting the bar for granting a patent is one of the crucial aspects for the functioning of a patent system, and represents a critical policy issue. On the one hand, too-loose requirement also allows inventions of little use or modest innovative value to be protected; on the other hand, however, a stringent requirement can prevent even substantial innovation from obtaining protection. In both cases, there is a risk of distorting the incentives to innovate, and therefore, undermining the correct functioning of the patent system.

Our analysis shows that the requirement for patentability may have an impact not only on firms' incentives to invest in R&D, but also on the direction of these investments. This issue seems to be particularly relevant in competitive settings, where firms compete both in the product market and at the R&D level. We show that the interplay between the strictness of the patent regime and firms' strategic motives is likely to influence firms' decisions regarding the direction of their innovative activities. To see this, we develop a model in which two competing firms first decide which of the two different technological areas to operate in, that is, firms choose the direction of their innovation, and then they invest in R&D in the chosen area. Once investments are made, firms compete in the product market. The innovation process is uncertain and, if successful, an innovator can protect its innovation from imitation by costlessly applying for a patent; protection is granted only if the innovation is sufficiently useful/nonobvious, i.e., if it satisfies the requirements imposed by patent law.

The firms' decision about the technological territory in which conducting R&D is a crucial feature of our model. When firms operate in the same technological area, their investments can give rise to technologically overlapping innovations. In this case, if both firms succeed, only one can be granted a patent. When firms operate in different areas, innovations and patents that protect them coexist. Crucially, when firms are active in the same technological area, an inefficient duplication of R&D may emerge, which makes differentiation in R&D trajectories desirable from a social efficiency perspective.

In this setting, we show that compared to a regime in which negligible innovations are patentable, stricter requirements for patentability may induce firms to invest more, thus positively affecting market efficiency. More importantly, we find that raising the bar for patentability may increase social welfare through its impact on the direction of firms' investments. While with a generous patent regime, firms prefer to invest in the same technological territory, stricter requirements for patentability induce firms to operate in different territories with desirable effects on efficiency. Hence, an appropriate determination of patentability requirements stimulates firms' innovative activities and eliminates any potential risk of wasteful duplication of R&D.

Our analysis was conducted in two parts. In the first part, we develop a very general reduced-form model of competition between two firms that, prior to competing in the product market, have to decide in which direction and how much to invest in R&D. This model is consistent with various setups and innovation modes; in the second part of the paper, for illustrative purposes we apply the analysis to two different settings: in the first one, firms invest in process innovations, while in the second one, firms invest in developing new products. These illustrative models will also allow us to add some interesting considerations regarding the applicability of our general theory.

The paper is organized as follows. In Section 2, we discuss how our analysis contributes to the economics literature. In Section 3, we present a general model of competing firms that must decide the direction of their R&D and how much to invest, prior to competition; this general model is then illustrated in Section 4 through a standard model of Cournot duopoly with process innovation (Section 4.1) and product innovation (Section 4.2). Section 5 concludes the paper.

2 Review of the literature

As mentioned, economics literature on the impact of patents on the direction of innovation is quite scant. To the best of our knowledge, the first study that deals with the relationship between patents and firms' technological trajectories is Von Graevenitz et al. (2013), where the authors present a model in which firms first choose the technological area of their R&D

activities and then decide how many patent applications to file. The main scope of this paper is on disentangling the effect of technological complexity on the increase in firms' patenting activities.

Bryan and Lemus (2017) analyzed the misallocation of R&D investments deriving from firms' incentives to pursue either easy and less valuable projects or complex ones that guarantee large value. They claimed that policies designed to influence the rate of innovation (such as patents and rewards) are not suitable for inducing an efficient direction of innovation. However, our analysis contributes to this debate from a different perspective by discussing how appropriate management of the requirements for patentability may affect the direction of firms' technological trajectories, thus avoiding inefficient duplication of R&D efforts. Similarly, Hopenhayn and Squintani (2021) analyzed the sources of the misallocation of R&D investments. They distinguished between static and dynamic sources of misallocation. The former arises from excessive entry in some technological areas and insufficient entry in others. The latter occurs because of the cost of reallocating research investments when a project is solved by one inventor. In our model, we found that innovators tend to concentrate R&D efforts on the same technological area, which induces them to invest less. However, we showed that policymakers can re-establish directional efficiency by increasing the cost of investing in overlapping technologies.[1]

Our contribution also adds to a recent paper by Comino and Manenti (2022). The authors studied how the presence of a stronger/weaker patent regime affects the technological trajectories of firms in a context in which firms strategically amass large patent portfolios. Interestingly, the authors find that in addition to the classical deadweight loss associated with the monopolistic position that they grant, patents can be the source of another potential inefficiency related to the distortion they may cause in relation to technology choices. In their analysis, the role of the patentability requirement was ignored, which is the focus of our study.

A study that is more closely related to ours is Chen et al. (2018). As in our study, the core of the analysis is the impact of patentability requirements on the characteristics of firms' R&D activities. The authors use a model of cumulative innovations to focus on the dynamic effects of nonobviousness, in which, in each period, firms engage in patent races for developing a new product that improves upon the current product. Firms choose between risky and safe

[1] Moraga-González et al. (2022) and Dijk et al. (2021) also investigated how firms allocate resources to different R&D projects; however, their focus is not on patents. More specifically, Moraga-González et al. (2022) focus on how mergers between two firms alter the direction of the R&D of the merged entity. Instead, Dijk et al. (2021) analyze a similar problem focusing on start-up acquisition.

project in each period. Stricter standards for patentability were found to have countervailing effects. On the one hand, it reduces the static incentives to invest in risky projects that may fail to lead to patentable innovations, on the other hand, it increases the dynamic incentives to invest in risky projects by extending the period of incumbency. Our study departs from Chen et al. (2018) in several directions, mostly because we want to see how the requirements for patentability impact firms' choices in the context of simultaneous innovations, whereby firms simultaneously compete in the product market and R&D activities.

Finally, our study contributes to the literature on the nonobviousness requirement, which is excellently reviewed in Denicolò (2008). Kou et al. (2013), in a similar setting to Chen et al. (2018), O'Donoghue (1998), and Hunt (1999) analyzed the impact of the nonobviousness requirement for patentability on firms' incentives to innovate; nonetheless, they do not discuss the impact of nonobviousness on the firms' technological choices, which is the focus of our paper.

3 A general framework

This section presents a general framework for discussing firms' R&D strategies in the presence of a more or less stringent patent regime. The goal is not to develop a "proposition-proof," fully-fledged model, but rather to discuss the conditions for the existence of certain equilibria in firms' choices to occur and to show that these conditions hold in fairly broad circumstances. To convince the reader, our framework will then be illustrated with the use of two specific theoretical models, one with firms investing in process innovations and one with firms investing in developing new products. Based on specific functional forms of firms' profits and R&D costs, we will be able to verify that the conditions of the general framework are satisfied. The general framework and illustrative models complement each other, and represent the two building blocks of our theory.

Our framework is general enough to encompass both the case of firms investing in gradual and drastic innovations, as well as in product and process innovations. For the sake of simplicity, in the rest of the paper we refer to product innovations.

3.1 The setup

Consider a market populated by two competing firms, i and j. Prior to competition, firms simultaneously innovate. The innovation process consists of the two following successive decisions: firms first decide the technological territory/project for conducting R&D, i.e., the direction of their innovation, and then how much to invest in the chosen direction. We assume that the decision to invest in a given technological territory is irreversible; for example, prior to conducting R&D in the chosen territory, firms must sink specific resources (i.e., acquisition of basic know-how and/or of specific machinery to conduct research/commercialize the innovation), which would be partly or entirely lost if the firm goes for a project in another technological area. These fixed-sum investments do not alter the subsequent decision on how much to innovate in the chosen technological area (i.e., how much a firm invests in R&D to develop the project). There are two possible technological trajectories along which firms can innovate, a or b. Once the trajectory is chosen, firms decide how much to invest in order to develop the project and then compete in the product market.[2]

It should be noted that we are completely agnostic on the value, either private or social, generated by the two projects. To eliminate obvious effects due to differences in the profitability of the two projects, we consider a perfectly symmetric context, in which the two trajectories are ex-ante identical both in terms of profitability and social value generated. Hence, let us indicate with $\beta_i > 0$ the value firm i R&D investments generates in the selected direction if it succeeds in innovating. β_i can be interpreted as the quality of the innovative product developed by the firm, in the case of product innovation, or the increase in production efficiency, in the case of process innovation. We also refer to β_i as the degree of firm i innovativeness. The amount of resources required to develop an innovation with a value of β_i is $I(\beta_i)$, with $I'(\beta_i) > 0$ and $I''(\beta_i) > 0$, regardless of the chosen technological territory. Finally, we assume that the R&D process is uncertain: once invested, firms may succeed in developing the innovation with a given probability $p \in (0,1)$; this probability is exogenous and, for the above reason, common to both firms and projects.[3]

Note that in our simplified setting, a firm chooses how much to invest in R&D, and given the exogenous probability p, these investments uniquely determine the degree of firm innovativeness, β_i. As a consequence, a firm can maximize profits by choosing either the amount of its investments, $I(\cdot)$, or its degree of innovativeness, β_i. For this reason, with a slight abuse of wording, we often refer to β_i as the level of investment in R&D chosen by firm i.

[2] It is possible to show that under mild conditions our results are also valid in a more general setting characterized by more than two technological areas.

[3] A natural extension of our setting, which we leave to future investigations, is to make the probability of success, p, endogenous, for example, by assuming that it increases with a firm investment in R&D.

The patent system. Without protection, a successful firm can be perfectly imitated by the rival; imitation is assumed to occur at no cost. To prevent imitation, the firm can apply to the patent office (PTO) to protect its invention. We assume that once an invention is developed, an innovator can costlessly apply to the PTO; consequently, whenever innovating, a firm always applies for a patent.[4]

An innovation is patentable if and only if its degree of innovativeness is sufficiently large. Formally, indicating with $\bar{\beta}$ the requirement for patentability, an innovation is patentable if $\beta_i \geq \bar{\beta}$. The larger $\bar{\beta}$, the more stringent the patent regime and the more useful/innovative the innovation must be in order to be patentable. If $\bar{\beta} = \varepsilon$, where ε is a positive and negligible number, then also marginal innovations can be patented. Throughout the paper, we assume that firms perfectly observe $\bar{\beta}$ when investing in R&D. Once uncertainty is resolved and the PTO has issued a patent, firms compete in the final market.

Hence, our model has the following three stages: at time $t = 1$, firms decide the direction of their R&D (either to invest in project a or b); then, in $t = 2$, firms decide how much to invest in R&D; finally, at time $t = 3$, once the uncertainty regarding innovations is resolved and the PTO has eventually issued the patent(s), firms compete in the product market. The solution concept is Subgame Perfect Nash Equilibrium.

Overlapping and nonoverlapping innovations. In the first stage of our game, firms decide in which direction to innovate (either technology area a or b). Depending on their technological choices, two cases can arise. In the first, firms choose to invest in the same technological area and to develop the same innovative project; in this case, if firms are successful and their innovations are eligible of being protected with a patent, they invest in *overlapping* innovations. Second, firms conduct R&D activities in different technological areas and invest in *nonoverlapping* innovations.

These two scenarios have a crucial difference in terms of patent protection; in the latter case, if firms succeed, they can both obtain a patent as their innovations fall in different technological domains; whereas in the former case, patents cannot coexist. Hence, when firms invest in the same territory and innovate successfully, only one of them can be granted a patent.

Examples of both scenarios abound. In the development of anti-covid vaccines, Astra-Zeneca and Pfizer hold distinct patents that protect their vaccines developed in nonoverlap-

[4] Imitation is crucial in our setting as the patent system would lose its justification in its absence. The assumption of full imitation is made without loss of generality; our results go through even in the presence of less-than-perfect imitation.

ping technological areas (viral vector *vs* mRNA vaccines). In shoe manufacturing, virtually each producer has its own patented system for cushioning soles (they differ for the underlining technology: foam, encapsulated air, shock absorbers, or other heel reinforcement pads). In both examples, firms compete in the final market with their patents, which, therefore, coexist as they are technologically different. The scenario with overlapping innovations is different, and it can be seen as a sort of race to patent, whereby innovators work independently of each other in the same technological area; if they both succeed in developing the project, only one is granted the patent. For instance, Nvidia and AMD, the two leaders in the production of graphic cards, have been working head-to-head in the development of ray-tracing and upscaling technologies, which are the new frontier in graphic cards. Ultimately Nvidia got better with its patented technology.

Noticeably, when both firms invest in the same technological area and successfully develop their invention, they apply for a patent whenever their technologies fulfill the requirement for patentability. As the two simultaneous innovations overlap, only one can be protected. For simplicity, we assume that the PTO randomly issues the patent; that is, when it receives two applications for patents belonging to the same technological territory, it issues a patent to each technology with a probability of 1/2.[5]

Firms gross profits. Under very mild conditions on the costs of R&D, which will be discussed below, it is possible to show that, unless the requirement for patentability is excessively stringent, at equilibrium, the two firms invest in R&D to obtain a patentable innovation if successful. As patenting occurs at no cost, a successful firm always applies for patent protection. Hence, three cases can potentially occur in the product market as follows: *i*) competition between two patent holders that occurs when both firms invest successfully in R&D and patent their innovations; *ii*) competition between one patent holder and one without the patent, when only one firm is successful; and *iii*) competition between firms without patents, when they both do not innovate. Clearly, in line with what we said before, the first scenario can only be realized if firms invest in nonoverlapping technologies. For each of these scenarios, let us indicate with:

i) $\pi_i(\beta_i, \beta_j) > 0$ the profit of firm i, gross of the cost of the R&D investments, when both firms hold a patent, where β_i and β_j are the qualities of firms innovations (we

[5]Given the symmetry of our setting, this assumption is consistent with the situation in which one firm may decide to apply for patent protection before the rival. As firms are identical, they do so simultaneously, and the PTO assigns the patent randomly. We thank Sarit Markovich for suggesting this interpretation.

sometimes refer to β_i and β_j as the quality of the patents);[6]

ii) $\pi_i(\beta_i, 0) > 0$ (resp. $\pi_i(0, \beta_j) \geq 0$) firm i's gross profits when only firm i (resp. firm j) holds a patent protecting its (resp. the rival's) innovation;

iii) $\pi_i(0, 0) \geq 0$ firm i's profits when neither firm holds a patent.

Given the probability of success in R&D, p, scenario i) occurs with probability p^2 and only when firms have invested in nonoverlapping territories, scenario ii) occurs with probability $p(1-p)$ and, finally, scenario iii) with probability $(1-p)^2$.

As mentioned above, our framework is symmetric; formally, this implies that for a given common level of firms' innovativeness, $\beta_i = \beta_j = \beta$, the profit functions satisfy the following conditions: $\pi_i(\beta, 0) = \pi_j(0, \beta)$, $\pi_i(\beta, \beta) = \pi_j(\beta, \beta)$, and $\pi_i(0, 0) = \pi_j(0, 0)$. Notably, our framework is sufficiently general to encompass various possible scenarios regarding the innovation process, whereby firms invest to develop gradual improvements in existing products or drastically innovate. In the former case, we have $\pi_i(0, \beta_j) > 0$, i.e., a firm that holds a patent still manages to stay in the market even in the case the rival has a patent, whereas in the latter case, we have $\pi_i(0, \beta_j) = \pi_j(\beta_i, 0) = 0$, i.e., a noninnovating firm is excluded from the market. If firms aim to develop completely new products, we have $\pi_i(0, 0) = \pi_i(0, \beta_j) = 0$, i.e., without a patent, the firm's profits are zero.

Basic assumptions. To obtain meaningful predictions of firms' investment decisions and their effects on social welfare, we assume that firms' profits and consumer surplus meet certain conditions. We do so in the least restrictive way, such that the setting is consistent with the most used models of oligopolistic competition.

The first natural assumption regarding firms' profits is that a firm's gross profit increases with the quality of its protected innovation and decreases with the quality of its rival innovation; formally: $\partial\pi_i(\beta_i, 0)/\partial\beta_i > 0$, $\partial\pi_i(\beta_i, \beta_j)/\partial\beta_i > 0$, $\partial\pi_i(0, \beta_j)/\partial\beta_j < 0$, and $\partial\pi_i(\beta_i, \beta_j)/\partial\beta_j < 0$. Additionally, we also make the following assumptions:

Assumption 1 (On profits ranking)**.** *For a given common quality of firm patents, β, firm i's profits are the highest when it is the only one holding a patent, and are the lowest when only the rival holds a patent; when both firms hold a patent, both firms obtain larger gross*

[6]Patent quality refers to the value generated by the underlying technology. We acknowledge that this could be misleading to a certain extent. In the literature, the term "quality" is usually referred to as the ability of a patent to meet statutory patentability requirements (Polk Wagner, 2009); following this definition, in our setting, all patents are of high quality as they are issued only if they meet the requirement $\bar{\beta}$.

profits than when they do not hold any patent. Formally, the following ranking in firms gross profits applies:

$$\pi_i(\beta,0) > \pi_i(\beta,\beta) > \pi(0,0) \geq \pi_i(0,\beta) \geq 0.$$

Assumption 2 (On marginal benefits)**.** *For a given common quality of firm patents, β, the marginal benefit that firm i enjoys from an increase in the quality of its protected innovation a) is larger when it is the only firm holding a patent than when its rival also holds a patent, but b) this marginal benefit is not too large; formally,*

$$a) \left.\frac{\partial\pi_i(\beta_i,0)}{\partial\beta_i}\right|_{\beta_i=\beta} > \left.\frac{\partial\pi_i(\beta_i,\beta)}{\partial\beta_i}\right|_{\beta_i=\beta}, \quad \textit{and} \quad b) \left.\frac{\partial\pi_i(\beta_i,0)}{\partial\beta_i}\right|_{\beta_i=\beta} < 2\left.\frac{\partial\pi_i(\beta_i,\beta)}{\partial\beta_i}\right|_{\beta_i=\beta}.$$

Naturally, how the quality of the innovations impacts on firms' profits depends on multiple factors, such as the competitive conditions of the market in which the firms operate, i.e., how innovations affect the degree of firms' product differentiation, and on the type of innovation, whether process or product innovation. Built on specific functional forms of market demand and firms profits, the illustrative models will allow us to discuss these specific aspects in greater details. Abstracting for the moment from these issues, Assumptions 1 and 2 simply imply that when a firm, is the only one holding a patent, is able to achieve a greater value from its patent. This is very reasonable and it is true both in absolute terms and at the margin.

We indicate the surplus enjoyed by consumers with $CS(\beta_i,\beta_j)$ when both firms hold a patent protecting innovations of quality β_i and β_j, with $CS(\beta_i,0)$ the surplus when only one firm holds a patent, and $CS(0,0)$ when no firm has developed technology and no patents are issued. It is reasonable to assume that the functions $CS(\cdot,\cdot)$ grow in both arguments. The more valuable the patented technologies, the greater the value for the consumers. Additionally, we also assume that

Assumption 3. *For a given common quality of firms' patents, β, the consumer surplus is the highest when both firms hold a patent and the smallest when neither firm holds a patent. Formally, the following ranking in consumer surplus applies:*[7]

$$CS(\beta,\beta) > CS(\beta,0) = CS(\beta,0) > CS(0,0) \geq 0.$$

Intuitively, innovations generate value; when both firms innovate and patent their inventions, a scenario that can occur only when firms invest in nonoverlapping technologies,

[7] $CS(0,0) = 0$ when firms invest in opening new markets, and neither firm succeeds in innovating; in this case, no value is created.

consumers enjoy greater benefits than when only one invention of a given quality is available.

As already stressed above, we expressly designed our setting so that there is no preferred direction of innovation from a private and social point of view, with the same degree of innovation; two innovations of the same degree of innovativeness but developed in different technological areas generate the same private and social value. Hence, from the social efficiency perspective, is the direction of R&D that matters; social efficiency is maximal when firms follow different technological trajectories and invest in nonoverlapping territories.

3.2 The equilibrium with a patent regime of maximum extension

Absent patent protection, the threat of imitation, reduces firms' incentives to invest in innovative projects and generates the so-called hold-up problem (Arrow, 1962). In the most extreme case, firms can be induced to refrain from R&D investments and market failure is maximal. Patents may help solve the hold-up problem by ensuring the appropriability of intellectual property. Our goal is to verify whether, by preventing imitation, the presence of a patent regime can solve this inefficiency. Specifically, we discuss how the presence of a more or less demanding patent regime in terms of requirements for patentability impacts firms' R&D strategies, both in terms of the amount of investments and their direction.

Given the characteristics of our framework, we can show that under very mild conditions, the equilibrium in firms' R&D activities has well-defined properties. In this section, we start with the case of a very extensive patent regime, in which an innovation can be protected regardless of its degree of innovativeness; formally, we have $\bar{\beta} = \varepsilon$, with ε arbitrarily small. In this setting, even almost obvious/useless innovations can be patented.

When firms invest in the same technological territory (either a or b) and both succeed in developing the innovation, they are both eligible to receive protection, but only one of them is granted a patent. The firm that does not receive the patent cannot bring its innovation to the market; therefore, it is as if it had not invested. Since with overlapping innovations the probability of a firm to receive the patent is equal to 1/2, the expected profits of firm i given firms R&D levels β_i and β_j, net of the R&D costs, are

$$E\Pi_i^o(\beta_i, \beta_j) = \frac{p^2}{2}(\pi_i(\beta_i, 0) + \pi_i(0, \beta_j)) + p(1-p)(\pi_i(\beta_i, 0) + \pi_i(0, \beta_j)) + (1-p)^2\pi(0, 0) - I(\beta_i), \quad (1)$$

with $i, j = 1, 2$, and where $\pi_i(\cdot, \cdot)$ are the gross profit functions in the various admissible scenarios, as previously discussed. The superscript ^{o} stays for *overlapping*. Firms choose β_i and β_j to maximize their expected profits. Assuming that $E\Pi_i^o(\beta_i, \beta_j)$ is well shaped, let us indicate with β^o the symmetric equilibrium level of firms R&D when they invest in the same technological territory.

When firms invest in different projects, they can both be granted a patent in case they succeed. Therefore, firm i's expected profits are

$$E\Pi_i^n(\beta_i, \beta_j) = p^2\pi_i(\beta_i, \beta_j) + p(1-p)(\pi_i(\beta_i, 0) + \pi_i(0, \beta_j)) + (1-p)^2\pi(0, 0) - I(\beta_i), \qquad (2)$$

and where the superscript ^{n} stays for *nonoverlapping*. Assuming that $E\Pi_i^n(\beta_i, \beta_j)$ is also well shaped, we indicate with β^n the symmetric equilibrium level of firms R&D in this case. It is possible to prove the following:[8]

Observation 1. *Suppose $\bar{\beta} = \varepsilon$. Firms' R&D investments are larger when they invest in nonoverlapping innovations: $\beta^n > \beta^o$.*

This is an important result in our setting and shows that when any innovation, even the most marginal ones, can be patented, firms invest more if they act in different technological areas. In fact, when firms invest in the same territory, there is a probability that, even if successful, a firm may not be able to patent its innovation; this occurs when the rival also succeeds and the PTO awards it the patent. Conversely, when firms invest in nonoverlapping technologies, a successful firm always gets the patent; everything else being equal, firms have more to gain from R&D compared to the case with overlapping technologies, which drives our result.

Let us now focus on consumers. It is possible to prove that

Observation 2. *Suppose $\bar{\beta} = \varepsilon$. Consumers are better off when firms conduct R&D in different technological territories.*

Firms invest more when they work on different projects; higher R&D investments translate into a higher value for consumers. Hence, it is no surprise that consumers are better off when they develop nonoverlapping technologies. In addition, with noncompeting technologies, there is a probability that two innovations are brought to the market at the same time, which generates even more value for consumers.

Imitation yields underinvestment in R&D; by preventing imitation, patents stimulate firm's investments. Therefore, a crucial aspect of evaluating the effectiveness of a patent system is to compare the level of innovation that firms are induced to generate in the presence of patents with the level that would be optimal from a social point of view. In our setting, it is

[8]The proofs of all the observations and remarks are relegated in the Appendix.

possible to show that even though a very generous patent system that allows any innovation to be patented stimulate firms' investments, it does not do so efficiently.

Observation 3. *Suppose $\bar{\beta} = \varepsilon$. Under mild conditions, firms underinvest in the social optimum.*

This observation provides a clear interpretation of the results. Even though the patent regime prevents imitation of any possible form of innovation, firms do not fully internalize the value they generate when innovating; hence, incentives to conduct R&D are suboptimal and firms are induced to invest an inefficient amount of resources.

Finally, let us focus on the technological choices firms make in the first stage of the game when they simultaneously decide which research project to conduct their R&D activity. It is easy to prove the following

Observation 4. *Suppose $\bar{\beta} = \varepsilon$. Under mild conditions in the R&D cost function, firms invest in overlapping innovations.*

This observation follows directly from Observation 1, and the fact that R&D costs increase with the degree of innovativeness. Therefore, given that $\beta^n > \beta^o$, when firms invest in the same technological area they bear a lower R&D cost: if the function $I(\beta)$ increases at a sufficiently large rate with β, the lower costs incurred in R&D more than compensate for the lower expected profits firms get when they invest in overlapping technologies.[9] Moreover, an additional reason why firms prefer overlapping to nonoverlapping innovations is that, as they invest less, when one firm does not succeed, it faces a less innovative rival with a beneficial effect on its expected profits.

From all these observations, it follows that a very generous patent system, although solving the classic underinvestment problem due to imitation and free-riding, presents several critical issues. First, as firms do not invest in a socially efficient amount of resources in R&D (Observation 3), incentives to invest are still scant. In addition, at the equilibrium, firms prefer to compete in the same territory (Observation 4), even though this option is not preferred by consumers (Observation 2) and, under mild conditions, it is also undesirable from the more general social welfare perspective.

Given the inefficiencies of the extensive patent regime, one may wonder whether a stricter patent regime where only sufficiently innovative technologies are patentable may induce firms to invest more and, possibly, in different technological areas.

[9] In our illustrative models, we show that a simple quadratic cost function is enough to lead to this result.

3.3 Nonnegligible requirement for patentability

Now, consider a stricter patent system, such that only innovations with a degree of innovativeness greater than $\bar{\beta} > 0$ can be patented.

From the previous section, we know that under an extensive patent regime $\bar{\beta} = \varepsilon$, firms invest β^o when they are active in the same technological territory, and β^n otherwise, with $\beta^o < \beta^n$. This implies that if $\bar{\beta} < \beta^o$, the patent regime is ineffective, as firms can still patent the inventions generated in the extensive patent regime. For stricter patent regimes, two following scenarios may occur: $\beta^c < \bar{\beta} \leq \beta^n$, i.e., the regime is binding only if firms invest in overlapping technologies, and $\bar{\beta} > \beta^n$, i.e., the regime is always binding. Hence, starting from β^o, as $\bar{\beta}$ becomes larger, firms must invest more to obtain patentable innovation, first in the case of investment in overlapping technologies, and then, for $\bar{\beta} > \beta^n$, in the case of investments in nonoverlapping technologies. In our setting, unless the patent system is too stringent, the equilibrium with nonnegligible and binding requirements for patentability, when firms invest in overlapping and nonoverlapping innovations, is characterized by firms investing $\bar{\beta}$. Hence, the following observation:

Observation 5. *When $\bar{\beta} > \beta^o$ (resp. β^n), firms active in the same territory (resp. in different territories) invest $\bar{\beta}$, provided that $\bar{\beta}$ is not too large.*

This observation is relevant from a policy perspective. This shows that a more stringent patent regime, in which only innovations with a degree of innovativeness greater than $\bar{\beta}$ are patentable, can stimulate firms' investments. Hence, a policymaker can design the requirements for patentability to reduce, possibly eliminate, the inefficiency highlighted in Observation 3.

It remains to be seen whether a more stringent patent regime affects firms' technological choices, inducing them to invest in different projects. Our last observation, which is the most relevant among all the observations obtained in our general setting, goes in this direction:

Observation 6. *For some common level of firms' innovation, the gross industrial profits of the investment costs are larger when both firms hold a patent than when only one firm holds a patent, formally if $\pi_i(\beta, \beta) + \pi_j(\beta, \beta) > \pi_i(\beta, 0) + \pi_j(\beta, 0)$, then there exists a level $\tilde{\beta}$ such that for $\bar{\beta} \geq \tilde{\beta}$ firms invest in different technological territories.*

With a mild requirement for patentability, as in the case $\bar{\beta} = \varepsilon$, firms operating in the same technological area tend to invest less than those operating in different areas. Given the convexity of the investment function, less investment means significantly lower costs;

this makes it preferable for firms to compete in R&D activities by choosing overlapping technological projects, even at the risk of failing to obtain a patent in case of success. When the requirement for patentability becomes more stringent, firms are induced to invest more in obtaining patentable innovation first in the case of overlapping and then in that of nonoverlapping innovations. This increases R&D costs and, given our assumptions on firms' gross profits, makes the choice of operating in the same technological area less advantageous compared to operating in different technological areas, to the point that firms may prefer the latter situation to the former.

[FIGURE 1 ABOUT HERE]

The message of our model is therefore clear: under relatively mild conditions of firms' gross profit functions, raising the bar of the requirements for the patentability of innovations can represent an effective tool for stimulating firms' investments and inducing differentiation in their technological choices (see Figure 1). It should be noted that inducing differentiation in firms' technological direction has the two following positive effects on market efficiency: if successful, firms generate more innovation, and the risk of duplicating R&D investments is eliminated. Hence, an adequate setting of the bar for patentability can be an effective tool for making the patent system more efficient. The illustrative models presented in the second part of our paper allow us to highlight all of this in more detail.

4 Illustrative models

Our general framework can be applied to different competition and innovation models. In this section, we complement this by solving two different illustrative models, each of which will allow us to highlight specific interplays between the type of innovation, the form of competition between firms, and the impact of patentability requirements on R&D strategies. In the first illustrative model, firms invest in process innovation and compete á la Cournot; in the second, firms invest R&D to develop a brand-new product and then compete.[10]

4.1 A model of process innovation

The timing is as above, in the first stage, firms choose between the two alternative projects a and b (i.e., firms decide the technological territory of their R&D). In the second stage, they

[10]The results hold under other competition models such as price competition with differentiated goods.

decide how much to invest and finally compete in the product market. More specifically, we assume that in the second stage, firms invest $I(\beta_k) = \beta_k^2/2$, $k = i, j$ to develop a process innovation that reduces their marginal cost of production from c to $c - \beta_k$, where c can be interpreted as current/preinnovation technology. Innovation is uncertain and, as before, we indicate with $p \in (0, 1)$ the probability of success in developing the innovation, which assumed to be the same for both projects.

Firms compete in quantities. Market demand is linear, $P(q_i, q_j) = a - q_i - q_j$, with $a > c > 4a/9$,[11] q_i and q_j are the levels of output produced by the two firms and P is the price. Given this framework, firms' second-stage profits, gross of R&D investment, in the three possible scenarios that can occur at the competition stage (both firms hold a patent, only one firm holds a patent, neither firm holds a patent) can be easily determined. Formally, if both firms hold a patent protecting a technology of quality β_i and β_j, gross profits of the R&D costs are

$$\pi_i(\beta_i, \beta_j) = \frac{(a - c + 2\beta_i - \beta_j)^2}{9}, \quad \text{and} \quad \pi_j(\beta_i, \beta_j) = \frac{(a - c + 2\beta_j - \beta_i)^2}{9}; \tag{3}$$

if only one firm holds a patent, let us say firm i, firms get

$$\pi_i(\beta_i, 0) = \frac{(a - c + 2\beta_i)^2}{9}, \quad \text{and} \quad \pi_j(\beta_i, 0) = \frac{(a - c - \beta_i)^2}{9}, \tag{4}$$

respectively. Finally, when neither firm holds a patent, they both get

$$\pi_i(0, 0) = \pi_j(0, 0) = \frac{(a - c)^2}{9}. \tag{5}$$

Given these payoffs, Assumptions 1 and 2 are satisfied.[12] With regard to consumer surplus, simple calculations show that the consumer surplus when both firms hold a patent and when only firm i holds a patent are:

$$CS(\beta_i, \beta_j) = \frac{(2(a - c) + \beta_i + \beta_j)^2}{18}, \quad \text{and} \quad CS(\beta_i, 0) = \frac{(2(a - c) + \beta_i)^2}{18},$$

Finally, if neither firm holds a patent, the consumer surplus is $CS(0, 0) = 2(a - c)^2/9$. Using

[11] The condition $a < 9c/4$ is necessary to ensure an internal solution in firms R&D choices.

[12] One can check that, for any given level $\beta_i = \beta_j = \beta$, Assumption 1 holds. Moreover, standard calculations reveal that $\partial\pi_i(\beta_i, 0)/\partial\beta_i|_{\beta_i=\beta} \equiv 4(a - c + 2\beta)/9 > 4(a - c + \beta)/9 \equiv \partial\pi_i(\beta_i, \beta)/\partial\beta_i|_{\beta_i=\beta}$; using these expressions, it immediately follows that $\partial\pi_i(\beta_i, 0)/\partial\beta_i|_{\beta_i=\beta} < 2\, \partial\pi_i(\beta_i, \beta)/\partial\beta_i|_{\beta_i=\beta}$. Assumption 2 is therefore satisfied.

these expressions, it is straightforward to observe that Assumption 3 is satisfied.

The equilibrium with an extensive patent regime. Consider a scenario characterized by an extremely generous patent system, such that negligible inventions are patentable. Given their decision on where to invest in R&D, firms choose β_i and β_j to maximize (1) and (2). Observations 1, 2, and 3 of our general theory are valid, and the following remark holds:[13]

Remark 1. *Assume that firms invest in process innovation and compete a' la Cournot with linear demand. When a generous patent regime is in place, i) R&D investments are higher under nonoverlapping than overlapping technologies, ii) firms do not invest efficiently, and iii) at the equilibrium, firms invest in the same technological territory.*

Points *ii*) and *iii*) highlight a misalignment between private and social incentives for R&D when the patent regime is particularly generous. On the one hand, independent of their technological choice, firms underinvest with respect to the social optimum. Underinvestment is due to firms failing to internalize the effects of innovations on consumers. When operating in the same and different technological areas, these effects are predominant, and failing to internalize them implies lower firms' investments in equilibrium. In addition to the underinvestment problem, firms allocate R&D efforts inefficiently, choosing to invest in the same project rather than in different projects, as social efficiency requires.

Nonnegligible requirement for patentability. Now, assume that $\bar{\beta} > 0$, that is, firms' innovations are patentable only if their degree of innovativeness is nonnegligible. In Remark 1, we show that when firms invest in process innovation, Observation 4 applies; hence, the following remark holds:

Remark 2. *Assume firms invest in process innovations and compete a' la Cournot with linear demand. When a stricter patent regime is in place, if R&D activities are sufficiently uncertain, that is, if $p \leq (7 - \sqrt{13})/4 (\approx 0.84)$, there exists a threshold for the patentability requirement $\tilde{\beta}$ such that for $\bar{\beta} > \tilde{\beta}$, firms choose to invest in different technological territories.*

This remark shows that unless R&D activities are almost certain ($p \leq 0.84$), there exists a minimum level of inventiveness, such that when the legal requirement is above this

[13]Note that in our general framework, we have only been able to characterize the consumer surplus (see Observation 2). However, in the illustrative models we know the functional forms of firms' profits and consumer surplus, therefore we can accurately evaluate the overall efficiency of the market with competing and noncompeting technologies.

threshold, firms' profits under overlapping technologies are lower than under nonoverlapping technologies, and firms are induced to invest in different technological territories. In other words, policymakers can adjust the strictness of the patent system as an instrument to induce firms to differentiate the direction of their technological choices. Figure 2 shows a graphical representation of Remark 2.[14] This figure is the correspondent of Figure 1 in the general theory. However, unlike the general theory, it shows that with process innovations, the strict requirement for patentability is ineffective when there is little uncertainty regarding the R&D process. In this case, firms invest in overlapping technologies regardless of their value of $\bar{\beta}$. Formally, when $p > (7 - \sqrt{13})/4$, the condition stated in Observation 6 never holds for any admissible value of $\bar{\beta}$.

[FIGURE 2 ABOUT HERE]

4.2 A model of product innovation

Now, consider the case in which the two firms invest in developing a new product, that is, in the case of success, a firm creates a brand-new market. The size of the market is increasing in β_i, which is the degree of innovativeness of firm i's product.

The timing is as follows: In the first stage, firms decide the technological territory of the innovation, and in the second stage, they invest in R&D. In the final stage, the firms compete in the product market. We proceeded by backward induction. As is clear now, when negligible innovations are patentable, three scenarios can occur at the competition stage: one firm holds a patent, both firms hold a patent, and no firm holds a patent. In the latter case, if no firm holds a patent because none has been successful in R&D or because none has invested, there is no innovation, and firms make zero profits:

$$\pi_i(0,0) = \pi_j(0,0) = 0.$$

When only one firm holds a patent, let us say firm i, a scenario that may occur when only firm i succeeds or when both firms succeed but only firm i is granted a patent as firms invested in overlapping technologies, we assume that the inverse demand for the new product is linear and equal to $P_i(q_i) = \sqrt{\beta_i} - q_i$; the degree of firm i's innovativeness β_i is a demand

[14] See the appendix for details regarding the various thresholds. As detailed, there is a limit on where to place the bar for patentability; if the regime is too strict, firms may prefer not to invest and produce with the current technology or to invest a smaller amount, and accept imitation by the rival in case of success in developing a nonpatentable innovation. This explains why for sufficiently high $\bar{\beta}$, firms' expected profit functions become flat and no longer depends on the requirement for patentability.

shifter: the greater β_i, the larger the demand. Firm i is a monopolist, while firm j, which does not hold any patents, does not earn profits. Solving firm i's maximization problem yields the following gross profit:

$$\pi_i(\beta_i, 0) = \beta_i/4, \quad \text{and} \quad \pi_j(\beta_i, 0) = 0. \tag{6}$$

Finally, when both firms hold a patent, a scenario that occurs when firms successfully invest in nonoverlapping projects, two innovations become available and compete in the product market. As before, we assume that the demand for innovation i is linear and it increases with β_i: $P_i(q_i, q_j) = \sqrt{\beta_i} - q_i - \gamma q_j$. The parameter $\gamma \in (0, 1)$ indicates the degree of substitutability between the two innovations: when γ is low, products are almost independent, whereas when γ approaches 1 products are homogeneous.[15] Note that, in this scenario, two patented innovations can be substitutes, even though they are obtained from different research projects.

For example, consider the development of COVID-19 vaccines in which pharmaceutical companies have developed various vaccines using different technical platforms (i.e., viral vector vaccines, gene vaccines, and inactivated vaccines). If developed, all vaccines perform the same function (they protect against the COVID infection) and can be seen as substitutes in the product market.[16] Firms decided on the technical platform with which to develop the vaccines (namely, the direction of their R&D) and then invested in research. Firms that have developed a vaccine using different technical platforms are granted a patent, whereas those that use the same platform engage a sort of patent race, with only one firm being granted a patent.[17]

Assuming that the two innovators compete in quantities, their gross profits of the R&D costs are as follows:

$$\pi_i(\beta_i, \beta_j) = \frac{(2\sqrt{\beta_i} - \gamma\sqrt{\beta_j})^2}{(4 - \gamma^2)^2}, \quad \text{and} \quad \pi_j(\beta_i, \beta_j) = \frac{(2\sqrt{\beta_j} - \gamma\sqrt{\beta_i})^2}{(4 - \gamma^2)^2}. \tag{7}$$

[15] The demand function can be easily generated from the net utility maximization of a representative consumer whose preferences over q_i and q_j are given by $U(q_1, q_2) = \sqrt{\beta_i} q_i + \sqrt{\beta_j} q_j - \left(q_i^2 + q_j^2 + 2\gamma q_i q_j\right)/2$ (see, Singh and Vives, 1984).

[16] This is true even though there might be substantial differences between vaccines in terms of dosage, type of adverse reactions, ease of conservation, and distribution

[17] In reality, Moderna and BioNTech/Pfizer, the two developers of mRNA vaccines, have both been granted a patent, but are now fighting a legal battle with allegations of patent infringement. In our model, we do not consider the possibility of the PTO issuing conflicting patents that protect overlapping inventions. However, we recognize that this represents a possible and interesting direction for extending our analysis.

Given these payoffs, it is observed that in this illustrative model with product innovations, Assumptions 1 and 2 are satisfied.[18]

Regarding the consumer surplus, it turns out that with product innovations, the consumer surplus when both firms hold a patent is

$$CS(\beta_i, \beta_j) \quad = \quad \frac{(\beta_i + \beta_j)\ (4 - 3\gamma^2) + 2\gamma^3\sqrt{\beta_i \beta_j}}{2\,(2-\gamma)^2\,(2+\gamma)^2},$$

when only firm i holds a patent is $CS(\beta_i, 0) = {}^{\beta_i}/_8$, and when neither firm holds a patent is $CS(0,0) = 0$. Using these expressions, it is possible to show that Assumption 3 is satisfied.[19]

The equilibrium with a generous patent regime. In the second stage, firms invest $I(\beta_k) = \beta_k^2/2$ in R&D, with $k = i, j$. If both firms invest in the same research territory, they will eventually develop overlapping technologies and only one of them will obtain a patent, which gives the patent holder a monopolistic position. If firms invest in different territories, they eventually develop nonoverlapping technologies that can be patented. In the latter case, the firms compete in the product market.

It is possible to verify that Observations 1, 2, and 3 in our general theory are valid. Moreover, Observation 4 confirms that competition is sufficiently intense when firms compete in the product market. Hence, the following remark applies:

Remark 3. *Assume that firms invest in product innovation. When a generous patent regime is in place, i) R&D investments are higher when firms are active in nonoverlapping technological territories, ii) firms do not invest efficiently, and iii) at the equilibrium, there exists a threshold of the degree of firms product differentiation $\hat{\gamma}(p)$ such that if $\gamma > \hat{\gamma}(p)$ firms invest in the same technological territory, with $\hat{\gamma}'(p) < 0$ and $\hat{\gamma}(0) = 2(\sqrt{2} - 1)$.*

Points *i*) and *ii*) replicate the points in Remark 1. As point *iii*) is concerned, it also goes along the same line as Remark 1, and if firms are active in different technological areas but any innovations they produce are in strong competition in the final market (i.e., the value of γ is high), firms prefer to invest in the same technological area and engage a patent race in the hope of becoming a monopolist.

[18] One can check that, for any given level $\beta_i = \beta_j = \beta$, $\pi_i(\beta_i, 0) > \pi_i(\beta_i, \beta_j)$, and $\pi_i(0,0) = \pi_i(0, \beta_j) = 0$; hence Assumption 1 holds. Moreover, standard calculations reveal that $\partial\pi_i(\beta_i, 0)/\partial\beta_i|_{\beta_i=\beta} \equiv 1/4 > 2/\left((2-\gamma)(2+\gamma)^2\right) \equiv \partial\pi_i(\beta_i, \beta)\partial\beta_i|_{\beta_i=\beta}$; using these expressions it immediately follows that $\partial\pi_i(\beta_i, 0)/\partial\beta_i|_{\beta_i=\beta} < 2\,\partial\pi_i(\beta_i, \beta)/\partial\beta_i|_{\beta_i=\beta}$. Assumption 2 is therefore satisfied.

[19] Formally, at $\beta_i = \beta_j = \beta$, $CS(\beta_i, \beta_j) - CS(\beta_i, 0)$ is equal to: $\beta(4 + 4\gamma - \gamma^2)/\left(8\,(2+\gamma)^2\right)$, which is positive for any $\gamma \in [0, 1]$.

Nonnegligible requirement for patentability. In contrast to the illustrative model with process innovation, Remark 3 shows that when firms invest in creating a brand-new product, they are more likely to follow different trajectories. Eventually, they may follow the same trajectory and invest in overlapping technologies when they are willing to avoid intense competition in the product market, i.e., when $\gamma > \hat{\gamma}(p)$. In this case, patent policy can once again play a role in inducing firms to follow different technological trajectories. Hence, assuming that $\gamma > \hat{\gamma}(p)$ (i.e., with $\bar{\beta} = \varepsilon$, firms invest in overlapping technologies), the following remark applies:

Remark 4. *Assume firms invest in product innovations. When a stricter patent regime is in place, if $\hat{\gamma}(p) < \gamma < 2(\sqrt{2} - 1)(\approx 0.83)$, there exists a threshold for patentability $\tilde{\beta}$ such that for $\bar{\beta} > \tilde{\beta}$ firms invest in different technological territories.*

This remark replicates Observation 6 for process innovation. This observation highlights that a threshold of the patentability requirement inducing firms to choose different technological trajectories exists if and only if $\pi_i(\beta, \beta) + \pi_j(\beta, \beta) > \pi_i(\beta, 0) + \pi_j(\beta, 0)$; that is, when for a given degree of innovativeness, β, the total gross producer surplus with two innovators competing in the product market is larger than when there is only one innovator. By using the gross profit functions (6) and (7) derived above, in this illustrative model this condition boils down to $\gamma < 2(\sqrt{2} - 1)$. When the two firms find themselves competing in the product market with innovative products that are not excessively substitutes, there is room for a policy intervention aimed at inducing firms to invest in different technological territories by setting a sufficiently stringent patentability requirement, which may have positive effects on social efficiency, as technological differentiation promotes a larger amount of innovation. Whereas, when innovations are strongly substituted ($\gamma > 2(\sqrt{2} - 1)$), there is no patent policy able to influence firms technological choices, and independent of $\bar{\beta}$, firms invest in the same territory.

5 Discussion and concluding remarks

In this paper, we propose a novel theory regarding the role of the requirement for the patentability of innovations. We show that this requirement may represent an effective policy instrument to reduce firms' underinvestment in R&D and crucially influence the direction of these investments. Absent patent protection, the threat of imitation reduces firms' incentives to invest in innovative projects; patents may help solve the underinvestment problem by ensuring the appropriability of intellectual property. However, in this study, we showed that in the context of competing firms simultaneously investing in innovation, an extremely generous patent system that allows negligible innovation to be patented is not enough to ensure an efficient level of investment. In addition, an extensive patent regime induces firms to engage in patent races by investing in overlapping technologies, thus generating further inefficiency owing to the associated duplication of R&D. Hence, an efficient patent system should not only stimulate firms to invest efficiently but also induce them to diversify their investments.

We showed that by strengthening the patentability requirement such that only innovations with a sufficiently high degree of innovativeness are considered for patent protection, the policymaker has two effects. Unless the requirement is not implausibly strict, the first direct effect is to mitigate firms' underinvestment in R&D. The second indirect effect is to alter firms' decisions about the direction of their R&D projects. If appropriately set, a stricter regime may induce firms to diversify the direction of their innovation investments.

In the second part of this study, we applied our general framework to two specific theoretical models. First, firms invest in process innovations aimed at reducing the cost of production; second, firms invest to produce a new product. We described the conditions under which a threshold for patentability induces firms to invest in different technological areas.

Our model is extremely simple, but can be extended in several directions. First, we do not consider the possibility for an innovator to license its invention to the rival firm.[20] Hence, a natural extension of our setting is to introduce technology licensing and evaluate the effects of various patent regimes on the direction of firms' R&D in this case.

Another extension worth studying is the introduction of R&D spillovers. In our setting, firms invest in R&D, but the benefit of the investment is entirely enjoyed by the firm that makes it. In reality, even in the presence of a patent system, investments in R&D can benefit third parties due to spillover effects. How the policy on patentability requirements changes in the presence of R&D spillovers is another interesting direction to investigate.[21]

[20]Licensing in oligopolistic market is a widely investigated topic (Sen and Tauman, 2018; Gallini and Wright, 1990; Kamien and Tauman, 1986; Katz and Shapiro, 1985). An extended body of research has studied the optimal contract scheme that an external producer of a superior technology should adopt to maximize the value of a patent (Colombo et al., 2023; Sen, 2005; Gallini and Winter, 1985). Others have analyzed the incentives of firms competing in the market to transfer their technology to rivals (Fan et al., 2018; Creane et al., 2013). Our model could be suitably modified to include licensing agreements and verify the interplay of patentability requirement and the existence of a market for patent licenses.

[21]If we assume that spillovers are stronger when firms invest in the same technological area, we expect that they may exacerbate underinvestment in overlapping technologies. Therefore, a policy aimed at inducing

Finally, governments/regulators employ common strategies to attract innovation and promote economic growth based on the appropriate design of their patent systems (see, Ernst & Young, 2022). Our analysis is one patent system/one PTO and, as such, it does not lend itself to analyzing competition between IP regimes; nonetheless, we believe that the presence of heterogeneous patent regimes with different requirements for patentability to attract innovators could be an interesting direction in which to extend our analysis. We leave this extension to future research.

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A Mathematical Appendix

A.1 General framework

Proof of Observation 1. By definition, the equilibrium levels of investments with overlapping and nonoverlapping technologies, β^o and β^n, must satisfy the following conditions:

$$\left(\frac{p^2}{2} + p(1-p)\right) \frac{\partial \pi(\beta_i, 0)}{\partial \beta_i}\bigg|_{\beta_i=\beta^o} = I'(\beta^o), \tag{A.1}$$

and

$$p^2 \frac{\partial \pi(\beta_i, \beta^n)}{\partial \beta_i}\bigg|_{\beta_i=\beta^n} + p(1-p)\frac{\partial \pi(\beta_i, 0)}{\partial \beta_i}\bigg|_{\beta_i=\beta^n} = I'(\beta^n). \tag{A.2}$$

Subtracting (A.1) from (A.2), the following condition must hold:

$$p^2 \left(\frac{\partial \pi(\beta_i, \beta^n)}{\partial \beta_i}\bigg|_{\beta_i=\beta^n} - \frac{1}{2}\frac{\partial \pi(\beta_i, 0)}{\partial \beta_i}\bigg|_{\beta_i=\beta^o} \right) + p(1-p) \left(\frac{\partial \pi(\beta_i, 0)}{\partial \beta_i}\bigg|_{\beta_i=\beta^n} - \frac{\partial \pi(\beta_i, 0)}{\partial \beta_i}\bigg|_{\beta_i=\beta^o} \right) = I'(\beta^n) - I'(\beta^o).$$

Provided that $\pi(\beta_i, 0)$ increases with β_i and that $I(\beta_i)$ is convex, given Assumption 2 this expression holds if and only if $\beta^n > \beta^o$. □

Proof of Observation 2. When firms invest in overlapping innovations and provided that, if successful, firm i (resp. j) develops innovation of quality β_i (resp. β_j), the expected consumer surplus is the weighted sum of the consumer surplus in various possible events:

$$ECS^o(\beta_i, \beta_j) = \frac{p^2}{2}\left(CS(\beta_i, 0) + CS(0, \beta_j)\right) + p(1-p)(CS(\beta_i, 0) + CS(0, \beta_j)) + (1-p)^2 CS(0,0).$$

Given the symmetry of our setting, when firms invest at the equilibrium level β^c, this expression boils down to the following:

$$ECS^c(\beta^o, \beta^o) = p^2 CS(\beta^o, 0) + 2p(1-p)CS(\beta^o, 0) + (1-p)^2 CS(0,0). \tag{A.3}$$

Now, considering the case with nonoverlapping innovations, the expected consumer surplus is as follows:

$$ECS^n(\beta_i, \beta_j) = p^2 CS(\beta_i, \beta_j) + p(1-p)(CS(\beta_i, 0) + CS(0, \beta_j)) + (1-p)^2 CS(0,0).$$

At the equilibrium level of investments, β^n, the above expression boils down to the following:

$$ECS^n(\beta^n, \beta^n) = p^2 CS(\beta^n, \beta^n) + 2p(1-p)CS(\beta^n, 0) + (1-p)^2 CS(0, 0). \tag{A.4}$$

As $\beta^n > \beta^o$ and given Assumption 3, one can see that $ECS^n(\beta^n, \beta^n) > ECS^o(\beta^c, \beta^o)$. □

Proof of Observation 3. Let us indicate as $EW^\ell(\beta_i, \beta_j)$, the social welfare in regime $\ell = o, n$ as a function of firms' investments, with $EW^\ell(\beta_i, \beta_j)$ assumed to be concave in β_i and β_j. Firms underinvest at the equilibrium if

$$dEW^\ell(\beta_i, \beta_j)\big|_{\beta_i=\beta^\ell, \beta_j=\beta^\ell} = \left(\frac{\partial EW^\ell(\beta_i, \beta_j)}{\partial \beta_i} d\beta_i + \frac{\partial EW^\ell(\beta_i, \beta_j)}{\partial \beta_j} d\beta_j\right)\bigg|_{\beta_i=\beta^\ell, \beta_j=\beta^\ell} > 0,$$

with $\ell = o, n$. Rearranging, this expression can be rewritten as follows:

$$\frac{d\beta_i}{d\beta_j} > -\frac{\partial EW^\ell(\beta_i, \beta_j)}{\partial \beta_j} \bigg/ \frac{\partial EW^\ell(\beta_i, \beta_j)}{\partial \beta_i}\bigg|_{\beta_i=\beta^\ell, \beta_j=\beta^\ell}.$$

Given the symmetry of our setting, the numerator and denominator of the right-hand side of this inequality are identical, and the underinvestment condition collapses to:

$$\frac{d\beta_i}{d\beta_j} > -1.$$

This condition indicates that firms invest less than the social optimum whenever their reaction functions in R&D choices have a slope greater than -1; this is a condition that is typically assumed to be satisfied in oligopoly models as it guarantees that the solution can be obtained as a convergence of a dynamic adjustment process (Martin, 1993). □

Proof of Observation 4. In the first stage, firms invest in overlapping technologies if $E\Pi^o(\beta^o) - E\Pi^n(\beta^n) > 0$ where, using expressions (1) and (2):

$$E\Pi_i^o(\beta^o) = \frac{p^2}{2}(\pi_i(\beta^o, 0) + \pi_i(0, \beta^o)) + p(1-p)(\pi_i(\beta^o, 0) + \pi_i(0, \beta^o)) + (1-p)^2 \pi(0, 0) - I(\beta^o),$$

and

$$E\Pi_i^n(\beta^n) = p^2 \pi_i(\beta^n, \beta^n) + p(1-p)(\pi_i(\beta^n, 0) + \pi_i(0, \beta^n)) + (1-p)^2 \pi(0, 0) - I(\beta^n).$$

One can see that $E\Pi^o(\beta^o) - E\Pi^n(\beta^n) > 0$ iff

$$I(\beta^n) - I(\beta^o) > \frac{p^2}{2}(2\pi(\beta^n, \beta^n) - (\pi(\beta^o, 0) + \pi(0, \beta^o))) \\ + p(1-p)(\pi(\beta^n, 0) - \pi(\beta^o, 0) + \pi(0, \beta^n) - \pi(0, \beta^o)),$$

where the left-hand side of this inequality is greater than zero as $\beta^n > \beta^o$ and $I'(\beta) > 0$. The above inequality is satisfied if $I(\beta)$ increases at a sufficiently large rate with β. □

Proof of Observation 5. For $\beta_i = \beta_j = \bar{\beta}$ to be an equilibrium, we need to check that firms cannot deviate profitably and unilaterally from $\bar{\beta}$. First, note that upward deviations cannot be profitable with $\beta_i = \beta_j = \bar{\beta}$, where $\bar{\beta}$ is greater than β^o (resp. β^n). Firms are already over-investing than their optimum, and the only possible deviation they might consider is investing below the threshold for patentability: $\beta^d < \bar{\beta}$.

We consider firm i (the same applies to firm j). When firms invest in overlapping technologies, their expected profits when they deviate are as follows:

$$E\Pi_i^{o,d} = p^2\pi(0, \bar{\beta}) + p(1-p)(\pi(\beta^d, \beta^d) + \pi(0, \bar{\beta})) + (1-p)^2\pi(0,0) - I(\beta^d); \quad \text{(A.5)}$$

Note that, having invested less than $\bar{\beta}$, when both firms succeed, only firm j gets the patent, and firm i gets $\pi(0, \bar{\beta})$, whereas when firm i succeeds, then firm j is free to imitate the unprotected innovation of firm i, and firm i gets $\pi(\beta^d, \beta^d)$.

When firms invest in nonoverlapping innovations, firm i profits from deviation are

$$E\Pi_i^{n,d} = p^2\pi(\beta^d, \bar{\beta} + \beta^d) + p(1-p)(\pi(\beta^d, \beta^d) + \pi(0, \bar{\beta})) + (1-p)^2\pi(0,0) - I(\beta^d), \quad \text{(A.6)}$$

where we assume that when both firms succeed, firm j can compete with a product that incorporates its invention and the unprotected innovation of the rival.[22]

From equations (1) and (2), we know that if firm i invests to comply with the requirements for patentability, it obtains

$$E\Pi_i^o(\bar{\beta}) = \frac{p^2}{2}(\pi_i(\bar{\beta}, 0) + \pi_i(0, \bar{\beta})) + p(1-p)(\pi_i(\bar{\beta}, 0) + \pi_i(0, \bar{\beta})) + (1-p)^2\pi(0,0) - I(\bar{\beta}),$$

[22]Hence, we are implicitly assuming that the two innovations are perfect complements and firm j can generate an innovation of overall quality $\bar{\beta} + \beta^d$. We make this assumption for simplicity, but our reasoning also works with less-than-perfect complementarity between firm innovations.

with overlapping technologies, and

$$E\Pi_i^n(\bar{\beta}) = p^2\pi_i(\bar{\beta},\bar{\beta}) + p(1-p)(\pi_i(\bar{\beta},0)+\pi_i(0,\bar{\beta})) + (1-p)^2\pi(0,0) - I(\bar{\beta}),$$

with nonoverlapping technologies. A downward deviation is not profitable if and only if $E\Pi^o(\bar{\beta}) - E\Pi^{o,d} > 0$ when firms are active in the same technological territory, and $E\Pi^n(\bar{\beta}) - E\Pi^{n,d} > 0$ when they invest in different territories. Using the expressions above, these conditions are satisfied if

$$\underbrace{I(\bar{\beta}) - I(\beta^d)}_{>0} \; < \; \frac{p^2}{2}\underbrace{(\pi_i(\bar{\beta},0) - \pi_i(0,\bar{\beta}))}_{>0} \; + p(1-p)\underbrace{(\pi_i(\bar{\beta},0) - \pi_i(\beta^d,\beta^d))}_{>0},$$

with overlapping technologies, and

$$\underbrace{I(\bar{\beta}) - I(\beta^d)}_{>0} \; < \; p^2\underbrace{(\pi_i(\bar{\beta},\bar{\beta}) - \pi_i(\beta^d,\bar{\beta}+\beta^d)}_{>0} \; + p(1-p)\underbrace{(\pi_i(\bar{\beta},0) - \pi_i(\beta^d,\beta^d))}_{>0}.$$

with nonoverlapping technologies. The left-hand side of these expressions represents the cost reductions induced by the lower investments in R&D, whereas the right-hand side shows the differences in the expected gross profits when firm i deviates, and they are positive by construction. This deviation is not profitable when the cost gains are lower than the expected gross profit losses.

In the case of a marginal deviation, $\beta^d = \bar{\beta} - \varepsilon$, these conditions hold as the cost gains are negligible, while the profit losses are of first-order magnitude. By continuity, the deviations in the neighborhood of $\bar{\beta} - \varepsilon$ are also not profitable. In general, deviations are not profitable, as long as $\bar{\beta}$ is not too large compared to β^d.[23] □

Proof of Observation 6. To prove this observation, it is useful to highlight the impact of a more stringent patent regime on firms' profits. As $\bar{\beta}$ increases, firms tend to invest more to comply with the stricter patentability requirement, which clearly reduces their profits compared to the $\bar{\beta} = \varepsilon$ regime. More specifically, when $\beta^o < \bar{\beta} \leq \beta^n$, the patent regime distorts firms' R&D choices only when they are active in the same technological territory, a further increase in the statutory requirement for patentability $\bar{\beta}$ reduces firms' profits even more. When $\bar{\beta} > \beta^n$, a further increase in $\bar{\beta}$ distorts firms' investments and profits in both scenarios (overlapping and nonoverlapping technological territories).

[23] In principle, other symmetric equilibria characterized by firms investing less than $\bar{\beta}$ cannot be excluded. In the case of multiple equilibria, we assume that firms select the equilibrium with higher R&D investments.

Formally, assuming that the patent system is not as stringent as inducing firms not to invest,[24] we can write this as follows:

$$\frac{\partial E\Pi^o(\bar{\beta},\bar{\beta})}{\partial\bar{\beta}} < 0 \quad \text{and} \quad \frac{\partial E\Pi^n(\bar{\beta},\bar{\beta})}{\partial\bar{\beta}} = 0, \quad \text{for} \quad \beta^o < \bar{\beta} \leq \beta^n \tag{A.7}$$

and

$$\frac{\partial E\Pi^o(\bar{\beta},\bar{\beta})}{\partial\bar{\beta}} < 0 \quad \text{and} \quad \frac{\partial E\Pi^n(\bar{\beta},\bar{\beta})}{\partial\bar{\beta}} < 0 \quad \text{for} \quad \bar{\beta} > \beta^n. \tag{A.8}$$

Hence, $E\Pi^o(\bar{\beta},\bar{\beta})$ and $E\Pi^n(\bar{\beta},\bar{\beta})$ are weakly monotonic decreasing functions, with $E\Pi^o(\bar{\beta},\bar{\beta}) > E\Pi^n(\bar{\beta},\bar{\beta})$ for $\bar{\beta} < \beta^o$; a necessary condition for the two firms to choose to invest in different technological areas being the existence of a level $\tilde{\beta}$ such that $E\Pi^n(\tilde{\beta},\tilde{\beta}) = E\Pi^o(\tilde{\beta},\tilde{\beta})$; if this happens, then for $\bar{\beta} > \tilde{\beta}$, $E\Pi^n(\bar{\beta},\bar{\beta}) > E\Pi^o(\bar{\beta},\bar{\beta})$, and firms choose different technological areas.[25]

If $\tilde{\beta}$ exists, two possible cases may occur, as shown in Figure 1. In case (a), $\tilde{\beta}$ lies between β^o and β^n; in case (b), $\tilde{\beta}$ is greater than β^n. Due to conditions (A.7), in case (a) the necessary and sufficient condition for an intersection is that $E\Pi^n(\beta^n,\beta^n) > E\Pi^o(\beta^n,\beta^n)$. Using expressions (1) and (2), this condition boils down as follows:

$$2\pi_i(\beta^n,\beta^n) > \pi_i(\beta^n,0) + \pi_i(0,\beta^n).$$

If this condition is verified, there exists a level of $\tilde{\beta} \in (\beta^o,\beta^n]$ such that

$$2\pi_i(\bar{\beta},\bar{\beta}) > \pi_i(\bar{\beta},0) + \pi_i(0,\bar{\beta}),$$

for any $\bar{\beta} > \tilde{\beta}$. In case (b), $\tilde{\beta} > \beta^n$, where $\tilde{\beta}$ solves $E\Pi^n(\tilde{\beta},\tilde{\beta}) = E\Pi^o(\tilde{\beta},\tilde{\beta})$. When this occurs then, for $\bar{\beta} > \tilde{\beta}$, $E\Pi^n(\bar{\beta},\bar{\beta}) > E\Pi^o(\bar{\beta},\bar{\beta})$. Using expressions (1) and (2), this condition boils again down to the following:

$$2\pi_i(\bar{\beta},\bar{\beta}) > \pi_i(\bar{\beta},0) + \pi_i(0,\bar{\beta}).$$

Hence, a necessary and sufficient condition for cases (a) and (b) to occur is that for some

[24] Intuitively, for a sufficiently high $\bar{\beta}$, one or both firms may decide not to invest, as it may be too costly to obtain patentable innovation. Therefore, we restrict our analysis by focusing on less extreme but more interesting patent regimes. Illustrative models will help us elaborate better on these extreme cases.

[25] In principle, it may happen that there exists more than one threshold $\tilde{\beta}$ such that $E\Pi^n(\tilde{\beta},\tilde{\beta}) = E\Pi^o(\tilde{\beta},\tilde{\beta})$. This does not alter our message, provided that there is at least one value of β for which this equality holds; when this occurs, there exists at least a value of $\bar{\beta}$ such that $E\Pi^n(\bar{\beta},\bar{\beta}) > E\Pi^o(\bar{\beta},\bar{\beta})$.

levels of firm innovation, $2\pi_i(\beta,\beta) > \pi_i(\beta,0)+\pi_i(0,\beta)$; in our symmetric setting, $\pi_i(\beta,\beta) = \pi_j(\beta,\beta)$ and $\pi_i(0,\beta) = \pi_j(\beta,0)$, from which the observation follows. □

A.2 Illustrative models

Proof of Remark 1. By substituting the gross profit functions (3), (4), and (5) back into expressions (1) and (2), we found that the expected profit functions with overlapping and nonoverlapping technologies are

$$E\Pi_i^o(\beta_i,\beta_j) = \left(\frac{p^2}{2}+p(1-p)\right)\left(\frac{(a-c+2\beta_i)^2}{9}+\frac{(a-c-\beta_j)^2}{9}\right)+(1-p)^2\frac{(a-c)^2}{9}-\frac{\beta_i^2}{2}, \tag{A.9}$$

and

$$E\Pi_i^n(\beta_i,\beta_j) = p^2\frac{(a-c+2\beta_i-\beta_j)^2}{9}+p(1-p)\left(\frac{(a-c+2\beta_i)^2}{9}+\frac{(a-c-\beta_j)^2}{9}\right)+(1-p)^2\frac{(a-c)^2}{9}-\frac{\beta_i^2}{2}, \tag{A.10}$$

respectively. Firms choose β_i and β_j to maximize these profits. Solving the system of f.o.c.s in the two scenarios, the symmetric equilibrium levels of innovation β^c and β^n can be obtained. It is easy to see that $\beta^n = 4p(a-c)/(9-8p+4p^2)$ and $\beta^o = 2(2-p)p(a-c)/(9-8p+4p^2)$, with $\beta^n > \beta^o$, consistently with Observation 1. This proves part i) of the remark.

We can now use β^o and β^n to evaluate the expected consumer surpluses when firms invest in the same and in different technological territories. Moreover, substituting $CS(\beta_i,\beta_j)$, $CS(\beta_i,0)$, and $CS(0,0)$ into expressions (A.3) and (A.4), we obtained

$$ECS^o(\beta^o,\beta^o) = \frac{2(81-144p+208p^2-192p^3+118p^4-42p^5+7p^6)(a-c)^2}{9(9-8p+4p^2)^2},$$

and

$$ECS^n(\beta^n,\beta^n) = \frac{2(81-144p+208p^2-120p^3+56p^4)(a-c)^2}{9(9-8p+4p^2)^2}.$$

It can be checked that $ECS^o < ECS^n \; \forall\, p \in (0,1)$, consistently with Observation 2.

Expected social welfare is the sum of the firm's expected profits and consumer surplus; hence, the expected social welfare in regime $\ell = o, c$ is $EW^\ell(\beta_i,\beta_j) = E\Pi_i^\ell(\beta_i,\beta_j) + E\Pi_j^\ell(\beta_i,\beta_j) + ECS^\ell(\beta_i,\beta_j)$. Using the profit functions given in expressions (1) and (2) and the consumer surplus functions in expressions (A.3) and (A.4), the expected social welfare conditional on β_i and β_j when firms invest in the same and different technological areas are

$$EW^o(\beta_i,\beta_j) = \frac{4(a-c)^2}{9}+\frac{2(2-p)p(a-c)(\beta_i+\beta_j)}{9}+\frac{11(2-p)p\left(\beta_i^2+\beta_j^2\right)}{36}-\frac{\beta_i^2+\beta_j^2}{2}, \tag{A.11}$$

and

$$EW^n(\beta_i,\beta_j) = \frac{4(a-c)^2}{9} + \frac{4p(a-c)(\beta_i+\beta_j)}{9} + \frac{p(11(\beta_i^2+\beta_j^2)-14\beta_i\beta_j p)}{18} - \frac{\beta_i^2+\beta_j^2}{2}, \quad \text{(A.12)}$$

respectively. Substituting β^o and β^n into these functions, the expected social welfare with overlapping and nonoverlapping technologies are therefore

$$EW^o(\beta^c,\beta^c) = \frac{2\left(5p^6-30p^5+110p^4-240p^3+344p^2-288p+162\right)(a-c)^2}{9\left(4p^2-8p+9\right)^2},$$

and

$$EW^n(\beta^n,\beta^n) = \frac{4\left(20p^4-84p^3+172p^2-144p+81\right)(a-c)^2}{9\left(4p^2-8p+9\right)^2}.$$

Simple calculations show that $EW^c(\beta^o,\beta^o) < EW^n(\beta^n,\beta^n)\ \forall\, p \in (0,1)$.

Expressions (A.11) and (A.12) are concave in β_i and β_j. The socially-optimal investment levels can be obtained by solving the first-order conditions for welfare maximization as follows:

$$\beta^o_w = \frac{(a-c)(4-p)p}{9-11p+4p^2}, \quad \text{and} \quad \beta^n_w = \frac{4(a-c)p}{9-11p+7p^2}.$$

It is possible to check that $\beta^o_w > \beta^o$ and $\beta^n_w > \beta^n$, consistently with Observation 3. This completes the proof of part *ii*) of the remark. Finally, we plug β^o and β^n in the expected profit functions and obtain the equilibrium firms' profits:

$$E\Pi^o(\beta^o,\beta^o) = \frac{(a-c)^2(81-144p+136p^2-48p^3-8p^4+12p^5-2p^6)}{9(9-8p+4p^2)^2}, \quad \text{(A.13)}$$

and

$$E\Pi^n(\beta^n,\beta^n) = \frac{(a-c)^2(81-144p+136p^2-48p^3-16p^4)}{9(9-8p+4p^2)^2}. \quad \text{(A.14)}$$

Simple algebra is enough to prove that $E\Pi^n(\beta^n,\beta^n) < E\Pi^o(\beta^o,\beta^o)$ for any $p \in (0,1)$, consistently with Observation 4. This concludes the proof of part *iii*) of the Remark. □

Proof of Remark 2. First, in order to prove the remark, we need to verify that firms do not have any incentives to unilaterally deviate from investing $\beta_i = \beta_j = \bar{\beta}$ (as in Observation 5). As in our general framework, we note that upward deviations cannot be profitable: with $\beta_i = \beta_j = \bar{\beta}$, where $\bar{\beta}$ greater than β^o (resp. β^n), firms are already over-investing compared to their optimum and the only possible deviation they might consider is to invest below the threshold for patentability: $\beta^d < \bar{\beta}$.

We consider firm i (the same applies to firm j). We used expressions (A.5) and (A.6) to derive the best response of the firm who decides not to adhere to the new patent requirements when the rival does so. Standard maximization yields the deviation investments:

$$\beta^{d,o} = \frac{2(1-p)p(a-c)}{9-2(1-p)p} < \beta^o, \qquad \text{and} \quad \beta^{d,n} = \frac{2p(a-c-p\,\beta)}{9-2p} < \beta^n.$$

These deviations are profitable if and only if they guarantee larger profits than complying with the patenting requirement, formally if $E\Pi_i^\ell(\beta^{d,\ell},\bar{\beta}) > E\Pi^\ell(\bar{\beta},\bar{\beta})$, with $\ell = o, n$. This happens when $\bar{\beta} > \hat{\beta}^\ell$ where $\hat{\beta}^\ell$ is the solution to the condition $E\Pi_i^\ell(\bar{\beta},\bar{\beta}) = E\Pi_i^\ell(\beta^{d,\ell},\bar{\beta})$. Using $\beta^{d,\ell}$ back into expressions (A.5) and (A.6) and solving the condition above, the threshold values of the patentability requirement that induce deviation are

$$\hat{\beta}^o = \left(\frac{3\sqrt{(9-2(1-p)p)(81-2(2-p)p(45-p(45-p(44-p(25-2(5-p)p)))))}}{(9-4(2-p)p)(9-2(1-p)p)} + 1\right)(a-c),$$

and

$$\hat{\beta}^n = \left(\frac{9-2(1-p)p}{9-2p(1-p^2)} + \frac{3\sqrt{(9-2p)(81-4p(45-p(63-4p(14-p(7-2p)))))}}{(9-4(2-p)p)\,(9-2p(1-p^2))}\right)(a-c).$$

Simple algebra proves that $\hat{\beta}^\ell > \beta^\ell$ for all $p \in (0,1)$. This result, which implies that firms are willing to increase the level of R&D to comply with a stricter patent requirement if is not too stringent, is consistent with Observation 5. Above these thresholds, at least one firm finds it profitable to deviate. Intuitively, as the patent regime becomes stricter, one or both firms will eventually decide not to invest to get a patent, as it would be too costly. This region of parameters is outside our area of interest, as a patent requirement approximating these values would simply be too stringent and, hence, extremely inefficient. We define the thresholds $\hat{\hat{\beta}}^l > \hat{\beta}^\ell$ with $\ell = o, n$ as the values of the patentability requirement that make both firms willing to stop investing $\bar{\beta}$ to get a patent. For such high values of $\bar{\beta}$ firms deviate by investing less; for the sake of simplicity, we assume that when $\bar{\beta}$ is this large, firms withhold R&D investments. This assumption implies a very marginal loss in generality, which exclusively concerns a region of the model in which $\bar{\beta}$ assumes very high and unrealistic values, but it allows us to simplify things consistently.

To complete the proof of the remark, we use the expressions of firms' gross profits given in (3) and (4) to verify the condition in Observation 6. The condition is satisfied for $\bar{\beta} \geq \tilde{\beta}$, where

$$\tilde{\beta} = \frac{(a-c)p\left((2-p)(9-4(2-p)p) + \sqrt{p(p(8p(p(97-2(8-p)p)-218)+2097)-1188)+324}\right)}{(9-4(2-p)p)(9-5(2-p)p)}.$$

Finally, from expressions (3) and (4) it is immediate to see that $\left|\partial^2\pi_i(\bar{\beta},\bar{\beta})/\partial\bar{\beta}^2\right| > \left|\partial^2\pi_i(\bar{\beta},0)/\partial\bar{\beta}^2\right|$, and a crossing between $E\Pi^o(\bar{\beta},\bar{\beta})$ and $E\Pi^n(\beta^n,\beta^n)$ exists if and only if $\tilde{\beta} \in (\beta^o,\beta^n)$; hence, if $E\Pi^o(\bar{\beta},\bar{\beta}) > E\Pi^n(\beta^n,\beta^n)$ for $\bar{\beta} = \beta^n$, then it is impossible that $E\Pi^o(\bar{\beta},\bar{\beta}) < E\Pi^n(\beta^n,\beta^n)$ for $\bar{\beta} > \beta^n$. In fact, because the policy introduces a distortion on the investment level of the firms, increasing the requirement for patentability lowers their payoffs (as the marginal costs exceed the marginal benefits). The analysis of the curvature of the profit function implies that the profits of firms investing in noncompeting technologies fall faster than those of firms investing in the same technological territory. Hence, a crossing can never occur when $\bar{\beta} > \beta^n$. Simple calculations reveal that $\tilde{\beta} < \beta^n$ when the probability of succeess in R&D is not too high — i.e., when $p < (7-\sqrt{13})/4$.

Figure 2 illustrates the remark in the two relevant cases, namely $p < (7-\sqrt{13})/4$ and $p > (7-\sqrt{13})/4$, where $(7-\sqrt{13})/4 \approx 0.84$. □

Proof of Remark 3. Substituting the gross profit functions (7) back into expressions (1) and (2), it turns out that the expected profit functions with overlapping and nonoverlapping technologies are

$$E\Pi_i^o(\beta_i,\beta_j) = \left(\frac{p^2}{2} + p(1-p)\right)\frac{\beta_i}{4} - \frac{\beta_i^2}{2}, \tag{A.15}$$

and

$$E\Pi_i^n(\beta_i,\beta_j) = p^2\,\frac{(2\sqrt{\beta_i}-\gamma\sqrt{\beta_j})^2}{(4-\gamma^2)^2} + p(1-p)\frac{\beta_i}{4} - \frac{\beta_i^2}{2}, \tag{A.16}$$

respectively. Firms choose β_i and β_j to maximize these profits. Solving the system of f.o.c.s in the two scenarios, the symmetric equilibrium levels of innovation are

$$\beta^o = \frac{(2-p)p}{8}, \quad \text{and} \quad \beta^n = \frac{p}{4}\left(1 - \frac{\gamma(4-\gamma(2+\gamma))p}{(2-\gamma)(2+\gamma)^2}\right).$$

It is easy to see that $\beta^n < \beta^n$ for any $\gamma \in (0,1)$ and $p \in (0,1)$. This proves part *i*) of the remark.

Using β^o and β^n into the consumer surplus functions $CS(\beta_i,\beta_j)$, $CS(\beta_i,0)$ occurring at the competition stage, and then substituting them back into expressions (A.3) and (A.4), the expected consumer surpluses when firms invest in the same and in different technological territories are

$$ECS^o(\beta^o, \beta^o) = \frac{1}{128}(p(2-p))^2,$$

and

$$ECS^n(\beta^n, \beta^n) = \frac{p^2\left((2+\gamma)^2 + (4+(4-\gamma)\gamma)p\right)\left(4-\gamma(\gamma(2+\gamma))(1-p)+8\right)}{32(2-\gamma)(2+\gamma)^4},$$

respectively. It is possible to check that $ECS^o < ECS^n \; \forall \, p, \gamma \in (0,1)$.

The expected social welfare is the sum of firm's expected profits and consumer surplus; using the profit functions given in expressions (1) and (2) and the consumer surplus functions in expressions (A.3) and (A.4), and substituting β^o and β^n into these functions, the expected social welfare when firms invest in the same and different technological areas are

$$EW^o(\beta^o, \beta^o) = \frac{3}{128}(p(2-p))^2,$$

and

$$EW^n(\beta^n, \beta^n) = \frac{p^2(\gamma(4-\gamma(2+\gamma))(1-p)+8)\left((\gamma(\gamma(3\gamma-2)-20)+8)p + 3(2-\gamma)(2+\gamma)^2\right)}{32(2-\gamma)^2(2+\gamma)^4},$$

respectively. Simple calculations show that $EW^o(\beta^o, \beta^o) < EW^n(\beta^n, \beta^n) \; \forall \, p, \gamma \in (0,1)$.

The socially-optimal investment levels can be obtained by solving the first-order conditions for welfare maximization:

$$\beta^o_w = \frac{5}{32}(2-p)p > \beta^o, \quad \text{and} \quad \beta^n_w = \frac{p\left(5(\gamma+2)^2 + (4-\gamma(5\gamma+12))p\right)}{16(\gamma+2)^2} > \beta^n.$$

This proves part *ii*) of the remark.

Finally, using β^o and β^n in expressions (A.15) and (A.16), and keeping in mind that firms invest in the same technological territory if and only if $E\Pi^o(\beta^o, \beta^o) > E\Pi^n(\beta^n, \beta^n)$, tedious calculations yield the threshold of product differentiation as a function of the probability of success p above which the condition holds. This threshold is defined as $\hat{\gamma}(p)$, which decreases monotonically in $p \in (0,1)$ and has the values $\hat{\gamma}(p) = 0 \approx 0.828$ and $\hat{\gamma}(1) \approx 0.649$. This proves part *iii*) of the remark. □

Proof of Remark 4. Using the payoffs derived above and applying the condition $\pi_i(\beta, \beta) +$

$\pi_j(\beta,\beta) > \pi_i(\beta,0) + \pi_j(\beta,0)$ in Observation 6, it is easy to show the following:

$$\frac{2\left((2-\gamma)\sqrt{\beta}\right)^2}{\left(4-\gamma^2\right)^2} - \frac{\beta}{4} > 0, \quad \text{if} \quad \gamma < 2(\sqrt{2}-1) \equiv \hat{\gamma}|_{p=0}.$$

Hence, if we are in $\gamma \in (\hat{\gamma}, 2(\sqrt{2}-1))$, it is possible to determanie a value $\tilde{\beta}$ such that if $\bar{\beta} > \tilde{\beta}$, firms invest in different technological territories; otherwise, they invest in overlapping technologies. Note that, as in the previous illustrative model, $\left|\frac{\partial^2 \pi_i(\bar{\beta},\bar{\beta})}{\partial \bar{\beta}^2}\right| > \left|\frac{\partial^2 \pi_i(\bar{\beta},0)}{\partial \bar{\beta}^2}\right|$. Hence, also in this scenario, a crossing point between profits exists only if $\tilde{\beta} \in (\beta^c, \beta^n)$. To ensure that no profitable deviations exist for firms investing in overlapping technologies, we calculate the payoff from the deviations in expression (A.5) and obtain the deviation investment level, which is written as $\beta^o_{dev} = p(1-p)/(2+\gamma)^2$. With this, we calculate the payoff from the deviation and compare it with the expected payoff of the firm when it complies with the new requirement $\bar{\beta} > \beta^o$. We find that a deviation is profitable if $\bar{\beta} > \hat{\beta}^o$ with:

$$\hat{\beta}^o = \frac{1}{8}\sqrt{\frac{p^2\left(4\gamma\left(\gamma^3+8\gamma^2+24\gamma+32\right)+\left(\gamma^4+8\gamma^3+24\gamma^2+32\gamma-48\right)p^2-4\left(\gamma^4+8\gamma^3+24\gamma^2+32\gamma-16\right)p\right)}{(\gamma+2)^4}} + \frac{1}{8}\left(2p-p^2\right).$$

Using simple algebra, it is possible to prove that $\beta^n < \hat{\beta}^o$, $\forall\, p, \gamma \in (0,1)$. Hence, no profitable deviations exist in the parameter region in which the policy can be implemented.

□

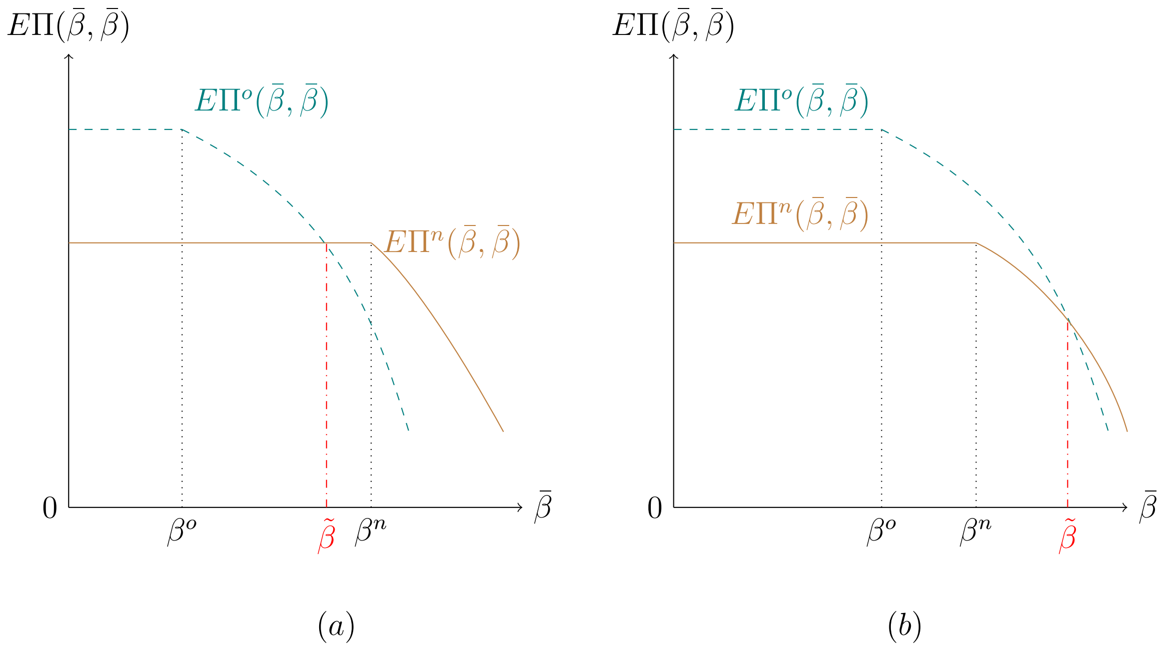

Figure 1: Effect of stricter patentability requirements on the direction of innovation. (a): $\beta^o < \bar\beta < \beta^n$; (b): $\bar\beta > \beta^n$.

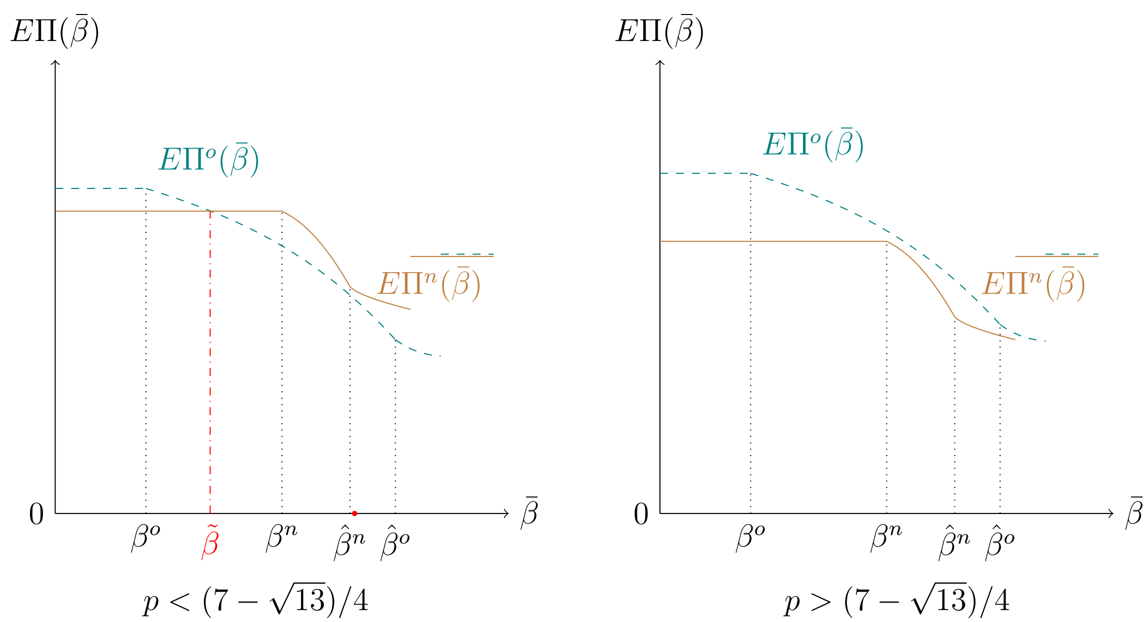

Figure 2: An illustration of Remark 2