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Price vs Market Share with Royalty Licensing: Incomplete Adoption of a Superior Technology with Heterogeneous Firms*

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Abstract

This article shows that the usual result of full adoption of a superior technology induced by pure royalty licensing may not hold when firms have different production technologies. By modeling a licensing game with an external innovator offering per-unit royalty contracts to downstream firms, this article shows that full adoption of the innovation occurs only if i) the new technology is sufficiently more efficient than the best one available in the market or ii) if the firms have similar efficiency levels. Moreover, I disentangle two distinct forces that influence the innovator's choice: a price effect (PE) and a market share effect (MSE). The former highlight the asymmetry in willingness to pay for the new technology. The inefficient firms, which benefit the most from the cost-reducing innovation, are willing to pay a higher price than their efficient rivals to become licensees. The latter illustrates the innovator's aim to maximize the volume of royalties collected by licensing to many firms. When PE dominates MSE, the patent holder sets a higher royalty rate and attracts fewer, less efficient firms. Otherwise, if MSE dominates, the patent holder lowers the royalty rate and attracts more firms to reach as many consumers as possible. From a policy perspective, I show that royalty licensing improves consumer surplus and that the positive effect increases with the number of licensees.

JEL Code: L13, L24, O31
Key-Words: Innovation, Licensing, Royalties, Price Effect, Market Share Effect

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1 Introduction

Article 28.2 of the Agreement on Trade Related Aspects of Intellectual Property Rights (TRIPs, 1994) states that "*patent owners shall also have the right to assign, or transfer by succession, the patent and to conclude licensing contracts*". Licensing a patent enables an innovator to simultaneously appropriate the knowledge she or he creates while guaranteeing its access to firms and subjects who generate value from exploiting that knowledge. Hence, from the innovator's perspective, the patent's value does not consist only of the quasi-rent granted by the temporary monopoly over the new creation. It also includes the value of the revenues derived from licensing it to other firms.[1]

This article investigates the economic incentives that drive the patent holder's choice of the optimal number of licensing contracts. It does so assuming an external patent holder offering a superior technology to downstream competing companies that are asymmetric in their marginal costs of producing the final good. It focuses on pure royalty licensing and shows that the patent owner faces a trade-off between the price of the technology and the volumes of royalties collected that drives her or his choice of how many contracts to sign. On the one side, licensing a technology to fewer inefficient firms guarantees the patent owner larger revenues per sale (intensive margin); on the other, including more firms at a lower price maximizes market penetration of the patented technology, increasing the count of royalties paid to the innovator (extensive margin).

I define the incentives to increase the intensive margin as *the Price Effect (PE)*, and the incentives to increase of the extensive margin as *the Marker Share Effect (MSE)*. Under the assumption of ex-ante symmetric firms, PE does not play a role. Technically, the royalty rate is constrained above by the value of the cost-saving effect enabled by the new technology, which, by construction, is the same for all adopting firms. Hence, the innovator can only leverage on volumes (i.e., the number of royalties collected) as the only source of revenue. Hence, from the innovator's standpoint, the only obvious choice is to maximize market penetration, and it does so by licensing the technology to all firms in the market. Instead, a more general approach that accounts for firms' heterogeneity

[1] Recent estimates by Neubig and Wunsch-Vincent (2017) indicate that between 1990 and 2015, royalty and licensing fees receipts and payments in the world economy grew at a rate of 10% per year. See also Arora et al. (2004) and Arora and Gambardella (2010) for estimates of the size of the technology market.

shows that PE is, in fact, an essential driver of the innovator's choice. Under some conditions, the innovator may prefer limiting the diffusion of her superior technology to just a subset of firms even with a pure linear per-unit royalty licensing scheme.

In other words, this paper challenges the robustness of one of the most established results in the literature, that when the patent holder chooses a per-unit royalty scheme, full adoption of the innovation emerges as the unique equilibrium (Sen and Tauman, 2018, Proposition 1, pg.42). It is not the first paper to do so, though. Exceptions to the "full diffusion" result can also be found in Lapan and Moschini (2000) and Sandrini (2022). The former demonstrates that, if the production of the final good requires more than one input, royalty licensing may imply partial adoption. More in detail, if the innovation, by altering the demand of the targeted input generates negative pressure on the prices of the other inputs, some firms may decide to keep the obsolete technology which has become cheaper. Instead, Sandrini (2022) shows how a licensor of a drastic innovation may lower the royalty rate and restrict the number of contracts to generate an asymmetry in the market. As a consequence, only some firms become licensees in equilibrium, whereas non-adopters exit the market. The main intuition is that when the cost-saving effect of the new technology is strong, the patent holder may prefer licensing the innovation to a subset of downstream manufacturers at a discounted price. The foreclosed firms that are no longer able to compete in the market exit, and the licensees expand their production to such an extent that the increased royalties collected by the patent holder more than compensate for the reduced royalty rate.[2]

This paper aims to find an alternative way to model the licensing game and the patent holder's maximization problem in a context where firms have different production technology. Indeed, the standard way of modeling process innovation as a linear parameter that lowers the marginal costs of production may not be suitable in this context, as it would imply that, in relative terms, ex-ante more efficient firms are able to reduce their already low costs to a much greater extent than inefficient ones. To overcome this issue, I model innovation as a technology that, if adopted by any

[2]Stamatopoulos and Tauman (2009) suggest that asymmetry in firms' marginal costs may lead to very different results than the homogeneous firms alternative model. More in detail, they analyze an asymmetric duopoly in which an external innovator licenses a cost-reducing technology. They find that, contrary to the case in which the firms are symmetric in marginal costs, in the asymmetric case the innovator prefers licensing the technology via a fixed fee rather than via an auction.

firm or group of firms, projects it or them to the technological frontier. To simplify it at most, suppose there are two firms, say i and j, competing in quantities of a homogeneous good. Their marginal costs are c_i and c_j, respectively. An external innovator licenses a non-drastic process innovation that allows the licensees to produce the final good at a cost $c^* < \min\{c_i, c_j\}$. Intuitively, the cost-reducing effect of the innovation differs between the two firms, as the less efficient one experiences a stronger cost-saving effect.

Within this model, this paper answers the following research questions. Does the patent holder license the innovation to all firms in the market? If not, which firm or group of firms does she target? What are the main drivers of this choice? Intuitively, each outcome implies a different effect on social welfare. Given the rival's production costs, providing an already efficient firm with even better technology is not the same as improving the least efficient firm's technology. Which outcome would the policy maker prefer? Are the social incentives aligned with the private ones? If so, under which conditions?

Using the example above, and assuming the firms' gross advantages from adopting the new technology are $c_i - c^*$ for firm i and $c_j - c^*$ for firm j. Assume $c_i < c_j$. It is apparent that firm j would benefit the most from the adoption. However, ex-post and given the royalty rate r, any adopting firm would produce at $c^* + r$, regardless of their ex-ante marginal cost. Considering that no firms would ever adopt the new technology if the net benefits are lower than the costs – i.e., if the price of the technology r exceeds its cost-saving effect - it is possible to state that the maximum willingness to pay is higher for the less efficient firm. Thus, the PE describes the incentives of the patent holder to exploit inefficient firms' large willingness to pay. Everything else being equal, she or he can charge a higher price for the same innovation to comparatively inefficient firms.

If the less efficient firm gets the new technology, it has to compete against a less but still efficient rival. On the contrary, if the efficient firm gets the innovation, it would be competing against a rival that is, in comparative terms, even less efficient than before. Thus, the market penetration of the new technology - i.e., the amount of output produced by means of the innovative process - is the highest if the ex-ante efficient firm gets the innovation, as her market share would be the most significant. The MSE describes the incentives of the patent holder to license the technology to firms that ensure the largest market coverage, so as to increase the number of royalty payments.

As revenues from per-unit royalty licensing depend on both the price and the amount of output produced with the technology, there is a well-defined trade-off between price and quantity. The dominance of either the PE or the MSE determines the innovator's choice on which firm (or group of firms) should receive the technology license.

Companies in labor-intensive sectors (such as call centers, logistics, and healthcare) continuously introduce new software and products to standardize production – e.g., Amazon wristband, collaborative robots, and the implementation of the manufacturing execution system (MES). For example, call centers heavily rely on AI-based software that evaluates workers' performance during calls. Software, such as Cogito Dialog, helps companies "establish better customer service solutions and new systems of employee and customer insights to improve internal analytics by leveraging unique behavioral and conversational data across your organization's business-critical systems" (quotation from the company's web page). Warehouses and large-scale retail companies use software and hardware to monitor and nudge workers. Companies with homeworking personnel use software, such as Time Doctor and StaffCop, to check workers' presence and productivity.[3] Many companies in these sectors introduce these software solutions to limit production inefficiencies and organize the workforce in a standard and measurable fashion. In other words, they move from their ex-ante particular production process (and the associated costs) and converge towards a standard procedure, eventually matching the cost structures of similar companies. Other examples include vertical industrial relationships in the semiconductor supply chain. In particular, one can think of a semiconductor architect, such as ARM Ltd., that sells the chip architecture to manufacturers – e.g., Apple, Intel, Qualcomm, Toshiba, and AMD, to mention some.[4] Finally, this analysis extends beyond the limited domain of patent licensing. Indeed, it applies also to sellers competing within the same product category in digital marketplaces – e.g., Amazon. Despite sharing the same cost of producing the homogeneous good, merchants face different costs of handling and delivering the parcel to final consumers. Alternatively, the platform offers a subscription-based service (e.g., Fulfillment by Amazon). By subscribing, the retailers outsource these final stages of the supply chain

[3]See Josh Dzieza, *The Verge*, February 27, 2020. See also Kevin Browse, *The New York Times*, June 23, 2019.
[4]I am grateful to Yusuke Ikuta for suggesting this illustrative example.

to a more efficient economic agent.[5]

The following paragraph summarizes the relevant literature. Section 2 provides a description and a detailed analysis of the model. Section 3 presents the main results and derives sufficient conditions for the main trade-off between price and market penetration to emerge. Section 3.1 applies the analysis to a market structure with n heterogeneous firms under the assumption of linear demand function. Section 4, analyzes the welfare implications of the results derived. Section 5 presents different extensions of the main model. In 5.1, I extend the main model to price competition with capacity constraints. In section 5.2, I assume that licensing occurs via ad-valorem fees. An example with linear demand and a duopolistic market is presented in section 5.2.1. Fixed fee licensing is briefly described in section 5.3. Finally, Section 6 concludes.

Review of the literature. Licensing under imperfect competition was first analysed by Kamien and Tauman (1986), Kamien et al. (1992), and Katz and Shapiro (1985). These early contributions seek to identify the most efficient licensing scheme. They suggest that upfront fees dominate royalties from the innovator's perspective, while the auction is the most efficient licensing scheme for an outside innovator. In light of these results, the dominance of royalties in empirical evidence has been considered puzzling.[6] A possible explanation has been advanced, among others, by Gallini and Wright (1990), who suggest that asymmetry of information can explain the dominance of royalties in empirical evidence. In contrast, Sen (2005) focuses on the technical constraints on the number of adopting firms. A non-exhaustive list of references on licensing under imperfect competition includes Erutku and Richelle (2007), Sen and Tauman (2007), Sen and Stamatopoulos (2016), Marshall and Parra (2019), and Parra (2019).[7]

Focusing on royalty licensing, Llobet and Padilla (2016) find that equilibrium prices under ad-

[5] Similarly, intermediaries in the advertising market may facilitate publishers to connect advertisers and consumers by lending their targeting technology (D'Annunzio and Russo, 2020). Also, in the energy market, demand aggregation may facilitate companies to lower their energy consumption costs.

[6] Rostoker (1984) shows that royalties (39%) and a combination of royalties and fees (49%) are largely more common than upfront fees (13%) in corporate-licensing transactions. Also, using Spanish data, Macho-Stadler et al. (1996) find a prevalence of linear contracts (83.6% of the sample), "in many cases degenerated, in the sense that they are based either on fixed fees or variable payments only" (pp. 46–47).

[7] See also Zou and Chen (2020) for an analysis of the optimal contract type with exclusive and non-exclusive licensing schemes and Ferreira et al. (2021) for a comparison of licensing outcomes under either quantity or price competition with horizontally differentiated products.

valorem royalties are lower than under per-unit royalties. Moreover, ad-valorem royalties benefit upstream innovators and do not necessarily hurt downstream producers. Hence, most licensing contracts include royalties based on the product's value.

Using a Cournot duopoly model, Hsu et al. (2019) argue that ad-valorem royalty licensing is superior to per-unit royalty licensing from the patent holder's perspective if the cost-reducing effect of the innovation is modest. Similarly, Fan et al. (2018) show that the profitability of different royalty licensing schemes depends on how effectively a patent holder can employ the new technology compared to the potential licensee. If the former is more efficient, then per-unit royalty licensing is superior to ad-valorem royalty licensing. Otherwise, ad-valorem royalty licensing is more advantageous.[8]

Most of these studies analyze the outcomes of a cost-reducing innovation when firms share a homogeneous technology that an invention can improve under a licensing agreement. As long as heterogeneity is considered, it is mainly intended as an output heterogeneity - i.e., product differentiation - not a technological one (Hernández-Murillo and Llobet, 2006). Notable exceptions are the contributions by Creane et al. (2013), Badia (2019), and Poddar et al. (2021). Creane et al. (2013) analyze technology transfer between heterogeneous firms. The authors investigate the welfare effects of technology transfer when a relatively more efficient firm offers its technology to a less efficient rival. Differently from the analysis provided in the present paper, Creane et al. (2013) focus on technology transfer between a firm that competes in the market for the final good to one of its rivals. Instead, Poddar et al. (2021) design a model of spatial price competition with two rivals endowed with different technology. They show that a pure royalty contract is the most efficient licensing policy from the outside innovator's perspective and that there is full diffusion of the technology. Here, I investigate the licensing outcome when the patent holder owns an innovation that improves the technological frontier and there are many companies competing in the market for the final good. Moreover, I highlight the underlying forces driving the inventor licensing decision (namely, the price effect and the market share effect), and show that partial

[8] A further challenge that firms face to participate in the technology market is how they should organize licensing. According to Arora et al. (2013) centralization is preferred when production-based incentives in the company are strong, whereas decentralization is chosen otherwise.

diffusion of the technology with a pure royalty contract is possible if the new technology is not excessively more efficient than the best available one. The result is robust to both per-unit and ad-valorem royalty licensing schemes.

Hernández-Murillo and Llobet (2006) is the paper that most closely relates to mine. The authors analyze process innovations with homogeneous firms offering differentiated goods and assume that the new technology is mostly fit to produce a specific product, but can also be employed by other firms with a loss of efficiency. They derive conditions for an optimal two-part tariff to be implemented as a combination of per-unit royalties and a fixed fee. Their theoretical framework is arguably more general than the one presented in this paper in that they allow for a larger family of licensing contracts. However, their focus is different as they analyze the efficient licensing of a patent holder, which entails a two-part tariff. Here, I am interested in showing that pure royalty licensing does not necessarily imply the full diffusion of the licensed technology, because of the underlying economic forces that generate the patent holder's problem of allocating licensing contracts.

Finally, Badia (2019) analyzes the (strict) conditions that allow an inefficient firm to catch up with a more efficient rival by adopting a superior technology. By modeling a licensing game where downstream heterogeneous firms bid for innovation, the author suggests that it is challenging for an inefficient firm to reduce the efficiency gap with its rival, primarily because of the asymmetry in market share implied by cost asymmetry. Similarly, by modeling a licensing game where the innovator sets per-unit royalties, this article shows that catch-up is more likely to occur if the market share difference is not too large relative to the difference in the willingness to pay.

2 The Model

Setup and assumptions. There is a continuum of firms of unit mass. Each firm i produces the same good with a technology that displays linear and constant marginal cost c_i. With a slight abuse of notation, I define technologies as the marginal cost of production implied by adopting said technology. Firms are technologically heterogeneous and technologies are uniformly distributed between $\underline{c}$ and $\bar{c}$, with $\underline{c} < \bar{c}$. Each technology is allocated to one firm, such that it is possible to rank firms in the technology domain. If firms i and j produce the good with technologies c_i and

c_j, respectively, then i is more efficient than j if $c_i < c_j$. Hence, the most efficient firm (firm 1) would be the one that produces with technology $\underline{c}$, whereas the least efficient one produces with technology $\bar{c}$. The rank can be written as $c_1 = \underline{c} < c_2 < ... < c_i < ... < \bar{c}$.

Firm i's production level is indicated by $q(c_i)$, with $\partial q(c_i)/\partial c_i < 0$. The total production level is $Q = \int_{\underline{c}}^{\bar{c}} q(c)\,\mathrm{d}c$. Moreover, I assume that all firms produce strictly positive output, i.e., $q(\bar{c}) > 0$. In other words, the most efficient firm has the largest share of the market, while the least efficient one has the smallest (but positive) market share. Because goods are assumed to be homogeneous, the above assumption requires a restriction on the market structure. More in detail, competition can not be in prices.[9] Therefore, in what follows, I assume firms compete in quantities and that firms face an inverse demand function $P(Q) > \bar{c}$.

I assume there is an outside innovator (she) who owns a patent for a technology that allows the firms to produce at a cost $c^* < \underline{c}$. All firms have a strictly positive willingness to pay for superior technology, as that is more efficient than the best one available in the market. For simplicity, I assume that technology adoption by any of the firms in the market will not lead the least efficient out of business (the innovation is strictly non-drastic). The licensing scheme is the per-unit royalties. In Section 5.2, I extend the analysis and show that the main results hold, with some minor differences, in the case of an ad-valorem royalty scheme. Finally, I briefly describe fixed fee licensing in section 5.3.

3 Main results and analysis

Exclusive dealing. First, let us consider the case in which the innovator sells the technology to one firm only, which means that the licensing contract includes an exclusivity agreement. The innovator decides which firm is most profitable to sell the technology to and at what price. I assume there is complete and perfect information, meaning that the innovator and the firms know exactly the ranking and allocation of all the technologies.

Intuitively, the maximum price a firm i is willing to pay is $r_i = c_i - c^*$, In words, the price of the new technology is constrained by the cost-saving effect it implies. Hence, the participation

[9] See section 5.1 for the analysis of price competition with capacity constraints.

constraint of firm i is $c^* + r_i \leq c_i$. Otherwise, the adoption represents a pure cost, and firm i rejects the offer. I define the royalty rate chosen by the innovator as $r = r_i(c_i)$, with $r'_{c_i} > 0$. The less efficient the targeted firm (c is high), the larger the cost-saving effect provided by the new technology and the less binding the participation constraint.

The innovator's maximization problem consists in selecting the right firm so that licensing revenues are maximized. Because each technology is allocated to only one firm and firms can be ranked in terms of technologies, the problem of the innovator can be written as follows:

$$\max_{c_i} \quad R^{ph} = r_i(c_i)\, q_i\left(c^*, r_i(c_i)\right)$$
$$\text{subject to} \quad r_i(c_i) \leq c_i - c^*$$

where the superscript *ph* stands for *patent holder.* The maximization problem yields the following first-order condition:

$$\frac{dR}{dc_i} = \underbrace{q_i\left(c^*, r_i(c_i)\right) \frac{d\,r_i}{d\,c_i}}_{\text{Price Effect}} \quad + \quad \underbrace{r_i(c_i)\left(\frac{\partial q_i}{\partial r_i}\frac{d\,r_i}{d\,c_i} + \frac{\partial q_i}{\partial c_i}\right)}_{\text{Market Share Effect}} = 0 \tag{1}$$

The first term of eq. 1 (price effect, PE hereafter) is positive, whereas the second one (market share effect, MSE hereafter) is negative. The PE identifies the increase in the maximum price that less efficient firms are willing to pay. As the price r cannot exceed the differential between the original cost function and the new one, an inefficient firm is willing to pay more for the latest technology, as her gains are comparatively high. Conversely, the MSE identifies how many units of the output will be produced with the new technology. Because less efficient firms have less strict participation constraints, they are willing to pay higher prices. In turn, this implies that inefficient firms would end up with a larger ex-post marginal cost of production than their more efficient rivals, by comparison.[10] In other words, a higher royalty rate comes with smaller volumes of output sold. Since the revenues from licensing depend on how many goods the licensee sells, the less efficient the adopting firm is, the lower the market penetration of the new technology.

[10] We should notice that, although the licensee would technically be the most efficient firm in the market, the net costs of production $c_i = c^* + r_i(c_i)$ are adjusted by the royalty rate, which is increasing in ex-ante marginal costs c_i.

The innovator chooses to license the firm whose c_i solves equation (1). Intuitively, there are two corner solutions to consider. First, if the MSE dominates the PE for all $c_i \in [\underline{c}, \bar{c}]$, the innovator sells the technology to the most efficient firm. Instead, if the PE dominates the MSE for all c_i, the innovator selects the least efficient firm. In every other case, there exists a firm i such that $\underline{c} < c_i < \bar{c}$ which is selected by the innovator as the only licensee. Formally, one can write

Lemma 1. *Define $\underline{c} - c^* = \epsilon$ and $q(\bar{c}) = \eta$, with both ϵ and η being arbitrarily small, non-negative numbers. Then,*

- *if $\epsilon \to 0$, the PE is strong. The patent holder has the incentives to offer the innovation to relatively less efficient firms;*

- *if $\eta \to 0$, the MSE is strong. The patent holder has the incentives to offer the innovation to relatively more efficient firms;*

- *if both $\epsilon \to 0$ and $\eta \to 0$, the innovator offers the innovation to a firm endowed with an intermediate technology.*

Proof. Consider the following ranking $c^* < \underline{c} < \bar{c}$ such that $q(\bar{c}) < q(\underline{c}) < q(c^*)$, i.e., all firms produce positive output.

First, assume the new technology is approximately the same as the best one available in the market ($c^* \approx \underline{c}$). In this case, the willingness to pay of the most efficient firm is very low, i.e., $r(\underline{c}) = \underline{c} - c^* = \epsilon$, with ϵ being an arbitrarily small (non-negative) number. As $\epsilon \to 0$, the willingness to pay of the most efficient firm converges to zero. Consequently, the revenues of the patent holder fall dramatically. In fact, $R^{ph}(\underline{c}) = \epsilon\, q(c^*, r(\underline{c})) \to 0$. The patent holder has a strict incentive to sell the license to a less efficient firm i at a price $r_i(c_i) > \epsilon$.

Second, assume the worst technology available $\bar{c}$ is such that $q(\bar{c}) = \eta$ with η being an arbitrarily small (non-negative) number. As $\eta \to 0$, the market share of the firm endowed with the worst technology converges to zero. Consequently, the revenues of the patent holder fall dramatically. In fact, $R^{ph}(\bar{c}) = r(\bar{c})\, \eta \to 0$. The patent holder has a strict incentive to sell the license to a more efficient firm i which produces $q(c^*, r_i(c_i)) > 0$.

Third, if both $\epsilon \to 0$ and $\eta \to 0$, the two opposite incentives lead the patent holder to select a firm i in the middle of the technological ranking. $\square$

Lemma 1 illustrates the sufficient condition for the existence of an interior solution. Intuitively, if both the most efficient firm is willing to pay a price close to zero, and the least efficient firm has almost no market share, the innovator prefers to select a firm in the middle of the ranking. Indeed, an intermediate firm is willing to pay more than the most efficient rival and produces more goods than the least efficient one.

Non-exclusive dealing. Let us now relax the assumption of exclusive licensing. The innovator can sell the technology to many firms. We assume that the royalty rate is public knowledge and that the innovator cannot negotiate the rate individually – i.e., no licensing fee discrimination. In other words, the royalty rate is a spot price, and the innovator cannot prevent firms from purchasing the right to use the innovation if they find it profitable to do so.[11]

Let us start from a case opposite the one analyzed above – i.e., when all firms get the license. In this case, the maximum price that the innovator can set is the price that the firms with the lowest willingness to pay would accept. The technology of the most efficient firm improves from $\underline{c}$ to c^*. Hence, the maximum price is the participation constraint $r(\underline{c}) = \underline{c} - c^*$.

From a welfare perspective, complete adoption of the superior technology would be optimal. In fact, after adoption all firms' costs of production boil down to $\underline{c}$. As the aggregate (and average) cost of production collapses, the total output increases. At the same time, as the royalty rate is fixed, less efficient firms would be the ones that gain the most from adoption. Market shares become equally distributed among all firms. Therefore, in terms of payoffs, efficient firms lose from adoption. One may wonder if, conditional on the royalty rate $r(\underline{c}) = \underline{c} - c^*$, complete adoption of the superior technology represents an equilibrium in the licensing subgame. As a matter of fact, efficient firms would see their efficiency quasi-rent zeroed out after all inefficient firms adopt the

[11] I am going to omit the case in which the patent holder can negotiate with a targeted and limited subset of companies, as it turns out to be suboptimal. Indeed, if a firm i purchases the innovation at a price $r(i)$, all the companies j with $c_j > c_i$ will find purchasing the license profitable. Because the patent holder would derive a larger volume of royalties for the same price, it is always optimal to allow as many licensees as possible. This intuition is formalized in Corollary 1

new technology. Therefore, one could expect that the most efficient firm rejects the offer to prevent such a scenario to occur. In turn, this may induce an unraveling process where all firms reject the innovator's offer. However, one can see that by setting $r(\underline{c}) = \underline{c} - c^*$, all firms but the most efficient have strict preferences for the new innovation, as they all improve their technology. The most efficient firm is just indifferent, but its choice doesn't affect the less efficient rivals' decision. Hence, by either assuming a tie-braking rule or adjusting $r(\underline{c}) = \underline{c} - c^* - \varepsilon$, with ε arbitrarily small and positive, all firms adopt the new technology.

The cases with full adoption and exclusive dealing with the least efficient firm are the two corner solutions. When the MSE dominates, the innovator licenses all downstream firms and maximizes market penetration. Instead, when PE dominates, the innovator sets the highest possible price and give the technology to the least efficient firm. Between these two extremes, the innovator can choose whatever firm i and set a price $r_i(c_i)$ such that all the firms with a technology less efficient than c_i are willing to pay the royalty rate. I define $m_i(c_i) \equiv (\bar{c} - c_i)$ as the number of firms with a marginal cost of production c_i or larger. If they adopt the new technology at a price $r_i(c_i)$, all the $m_i(c_i)$ licensees produce $q(c^*, r(c_i))$. Then, the problem of the innovator writes as:

$$\begin{aligned} \max_{c_i} \quad & R^{ph} = r_i(c_i)\, m_i(c_i)\, q(c^*, r(c_i)) \\ \text{subject to} \quad & r_i(c_i) \leq c_i - c^* \end{aligned}$$

The maximization problem yields the following first-order condition:

$$\frac{dR}{dc_i} = \underbrace{m_i(c_i)\, q(c^*, r(c_i))\, \frac{d\, r_i}{d\, c_i}}_{\text{Price Effect}} \quad + \quad \underbrace{r_i(c_i) \left(q(c^*, r(c_i)) \frac{\partial\, m_i}{\partial\, c_i} + m_i(c_i) \left(\frac{\partial\, q}{\partial\, c_i} + \frac{\partial q}{\partial ri} \frac{d\, r_i}{d\, c_i} \right) \right)}_{\text{Market Share Effect}} = 0 \quad (2)$$

As in the case of exclusive dealing, the patent holder chooses c_i such that equation (2) is satisfied. Intuitively, two corner solutions may emerge: first, if equation (2) is negative for all $c_i \in [\underline{c}, \bar{c}]$, then the price of the technology would be $r(\underline{c})$ so that all firms have access to it. Conversely, if equation (2) is positive for all $c_i \in [\underline{c}, \bar{c}]$, then only the least efficient firm will receive the technology, which means that the price is $r(\bar{c})$. In the former case, the MSE always dominates the PE, while the opposite is true in the latter case. If equation (2) is satisfied for some $c_i^* \in [\underline{c}, \bar{c}]$, then the two forces compensate each other and there is an interior solution for which the price of the technology is $r(c_i^*)$, and $(\bar{c} - c_i^*)$ firms adopt the technology.

Proposition 1. *From Lemma 1, assume $\epsilon \to 0$ and $\eta \to 0$ hold. Then, royalty licensing results in the incomplete adoption of superior technology.*

Proof. The proof to Proposition 1 stems from the same logic as the prof to Lemma 1.

First, assume $\epsilon \to 0$. Then, the new technology does not really improve the technology of the most efficient firm. Eventually, its willingness to pay for it is approximately zero, which limit the revenue from licensing regardless of the volume of royalties collected.

Second, assume $\eta \to 0$. Given the technology c^* and the royalty rate $r(\bar{c})$, the units produced by the least efficient firm are approximately zero, and so is the licensing revenue of the patent holder.

Third, if both $\epsilon \to 0$ and $\eta \to 0$, there exist a company i associated to a technology c_i such that the patent holder has no incentives to license the innovation to all the firms or to only the least efficient one. Instead, she offers the new technology to a subset of firms that include all companies ranked i or worse. $\square$

Corollary 1. *The patent holder always prefers to offer non-exclusive contracts.*

Corollary 1 derives from simple arithmetic. The price $r_i(c_i) \leq c_i - c^*$ that the patent holder has to set to attract firm i is the same regardless of the legal obligation (exclusivity clauses). If the patent holder is not legally obliged to supply only one company with the innovation, then she could sell it to all the firms less efficient than i at the same price. Even if i's production shrunk as the best response to the fall in rivals' average marginal costs, the total market penetration of the innovation would be strictly larger in the non-exclusive deal. Hence, because the patent holder can collect larger volumes of the same royalty rate, she prefers non-exclusive agreements to exclusive ones.

3.1 Example: Oligopoly with linear demand function

In this subsection, I provide an example of the general results derived above by means of a discrete model with n oligopolistic firms facing a linear inverse demand function $P = A - Q$, where $Q = \sum_{i=1}^{n} q_i$. Assume each company produces the homogeneous final good at a marginal cost $c(i)$, such that $c_1 < c_2 < ... < c_i < ... < c_n$. Accordingly, the final output can be ordered on (i) as follows: $q_1 > q_2 > ... > q_i > ... > q_n$, where

$$q_i = \frac{A - nc_i + \sum_{j \neq i} c_j}{n+1}.$$

The patent holder owns a technology that allows firms to produce the good at a marginal cost $c^* < c_1$.

To keep the analysis short, I will focus on the scenario where the patent holder offers non-exclusive dealing, and all the participation constraints of the firms are binding – i.e., the patent holder cannot set the profit-maximizing royalty rate.

We know from the analysis above that for a given price of technology r, if r satisfies the participation constraint of firm i, it also satisfies the participation constraint of all the companies with a less efficient technology (higher marginal cost of production). In other words, if the patent holder sets a price $r(i) \leq c_i - c^*$, all companies $j \neq i$ such that $c_j > c_i$ will also purchase the license. As before, I assume there is no scope for price discrimination.

The patent holder selects (i) to maximize her objective function:

$$R^u = (n - i + 1)\, Q(c_i, c_j)\, (c_i - c^*) \tag{3}$$

Where $(n-i+1)$ is the number of adopting firms, $c_i - c^*$ is the maximum price for the license such that firm i is indifferent between buying or not, and $Q(c_i, c_j) = \left(A - (i+1)c_i + \sum_{j=1}^{i-1} c_j\right)/(n+1)$ is the associated output level of the adopting companies, given their updated marginal costs $c^* + r(i) = c^* + c_i - c^* = c_i$.

Because c_i increases in i, the less efficient the targeted firm is, the larger the price of the technology that the patent holder can charge. However, a smaller number of firms will adopt the technology. This relationship illustrates the trade-off between the price effect and the market share effect explained above.

From equation (3), the maximization problem of the patent holder yields:

$$i^* = \arg\max_i \{(n-i+1)\, Q(c_i, c_j)\, (c_i - c^*)\}$$

Moreover, the equilibrium i is chosen so that the patent holder cannot derive higher profits by licensing any other group of companies. Formally, this condition requires:

$$(n-i+1)\, Q(c_i, c_j)\, (c_i - c^*) > (n-k+1)\, Q(c_k, c_j)\, (c_k - c^*)$$

with $k \neq i$ and $Q(c_k, c_j) = \left(A - (k+1)c_k + \sum_{j=1}^{k-1} c_j\right)/(n+1)$. Hence, it is possible to derive the following proposition

Proposition 2. *The patent holder decides to license the new technology to companies that are less or equally efficient than c_i, where c_i solves $\frac{c_i - c^*}{c_k - c^*} \geq \frac{(n-k+1)Q(c_k,c_j)}{(n-i+1)Q(c_i,c_j)}$ $\forall\ k \neq i$.*

An analytical solution proves difficult without some restrictions on the number of firms in the market and the distribution of marginal costs among them. However, it is possible to show numerically the existence of an interior solution. Assume $A = 20$, $n = 10$ and $c(i)$ are ordered from $c_1 = 1.1$ to $c_{10} = 2$ by a 0.1 increase ($c_2 = 1.2, c_3 = 1.3, ...$). Table 1 illustrates the results in different scenarios, in which the new technology enables the production of the final good at a cost that varies between $c^* = 1$ and $c^* = 0.5$.[12] For low values of c^*, the PE is weak, as the maximum price charged by the patent holder is always large, regardless of the company selected. Thus, when c^* is low, everything else being equal, the patent holder licenses her technology to a larger number of companies. Vice versa, when c^* is large, the patent holder exploits the asymmetry of efficiency between companies to maximize the licensing revenues. The patent holder may decide to lower the innovation diffusion and supply a small number of companies by charging a higher price.

[12] Notice that, with these assumptions, i) the profit-maximizing licensing price $(A - c^*)/2$ exceeds the maximum price that satisfies the participation constraints of all the firms, ii) all firms are active in the market, and iii) the innovation is never drastic.

Table 1: The optimal choice of the subset $\{i, ..., n\}$ of licensees by the patent holder. Entries represent the patent holder's licensing revenues for any pair (i, c^*), given the parameter values specified in the main text.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$
$c^*=1$	1.7181	3.0600	4.0146	4.5818	*4.7727*	4.6091	4.1236	3.3600	2.3727	1.2273
$c^*=0.8$	5.1545	6.1200	6.6909	*6.8727*	6.6818	6.1454	5.3018	4.2000	2.9000	1.4727
$c^*=0.6$	8.5909	9.1800	*9.3673*	9.1636	8.5909	7.6818	6.4800	5.0400	3.4273	1.7182
$c^*=0.5$	10.3091	*10.7100*	10.7055	10.3091	9.5455	8.4500	7.0691	5.4600	3.6909	1.8409

4 Welfare Analysis

The previous section discusses the private incentives of a patent holder to license her innovation to a subset of firms competing in the downstream sector. Here, I focus on the social welfare implication of the rationing of licensing derived above. The literature has stressed the ambiguous effects of licensing on social welfare and consumer surplus. In particular, using per-unit linear royalty, licensing may represent a device through which companies coordinate and collude to increase their prices (Mukherjee, 2005; Faulí-Oller and Sandonís, 2002). Those results are derived from modeling licensing as a technology transfer from an innovative firm to its competing rivals. In other words, licensing is a horizontal market relation.

When looking at the problem from a vertical perspective – i.e., when an outside innovator licenses her technology to firms competing in a downstream sector, the ambiguity shrinks. Indeed, licensing a process innovation is supposedly always (weakly) welfare-improving. The result of this analysis is no exception. However, there are some crucial differences that it is worth mentioning. Let us focus on licensing via per-unit linear royalties. The literature has mainly focused on licensing a process innovation to homogeneous companies (in terms of production efficiency). Hence, because of the constrained maximization problem of the patent holder, the price of the technology absorbs all the cost-reducing effects of the innovation, leaving the price unaltered. Welfare improves because some production costs become surplus and are extracted by the patent holder, whereas consumers do not generally benefit from introducing a more efficient technology.[13] However, if one allows

[13]This analysis assumes the patent holder possesses full bargaining power. Indeed, if the licensees retain some

downstream companies to be heterogeneous in their efficiency levels, even assuming the patent holder has full bargaining power, the welfare effect of licensing tends to be more beneficial to consumers. To show that, let us focus on non-exclusive licensing in a model as the one presented in section 2. Consider the case in which the innovation is adopted by a subset of firms $m = \bar{c} - c_m$ at a price $r_m(c_m) = c_m - c^*$ such that all the firms whose cost of production is larger or equal than c_m buy the license. The new total output is $Q = \int_{\underline{c}}^{c_m} q(c)dc + (\bar{c} - c_m)q(c_m)$. Because $(\bar{c} - c_m)q(c_m) > \int_{c_m}^{\bar{c}} q(c)dc$, total output expands as a consequence of licensing and price falls.[14] Hence, consumers benefit from the introduction of innovation. From the social planner's standpoint, a full diffusion of innovation would imply the largest expansion of consumers' surplus.

Licensing fee discrimination. In the analysis presented above, licensing fee discrimination has been ruled out by assumption. Indeed, if the patent holder could price discriminate the downstream firms, she would offer each of them a royalty rate that matches their willingness to pay. The licensing game would obviously result in full diffusion of the superior technology as the PE would not play any role. However, because the patent holder extracts all the cost-saving effects of the innovation via the personalized royalty rate, the effect of innovation diffusion on consumer surplus would be nil. As in the case of homogeneous firms, welfare improves because some costs would become surplus extracted by the patent holder, but the price of the final good would not fall. Hence, somehow paradoxically, a consumer-oriented policy maker would be indifferent between a market in which the innovation is not licensed at all and one in which there is full diffusion of innovation, but the patent holder can price discriminate downstream firms.

5 Extensions

In this section, I will modify some of the main assumptions of the model presented in sections 2 and 3. First thing, I will illustrate the results assuming downstream firms are capacity constrained so

of the surplus extracted by the patent holder, the final good's price would fall, and consumers would be better off. However, the analysis in this section holds for any given level of the patent holder's bargaining power.

[14] One may argue that ex-post – because non-adopting firms shrink their production level as the best response against relatively less inefficient rivals – the effect of licensing on output is ambiguous. However, because the average cost of production is lower after the adoption of the technology by m firms at the price r_m, total output is expected to expand.

that the demand cannot be satisfied by only one supplier. This framework implies firms engage in marginal pricing strategy, which is common in energy markets, in particular the day-ahead market for electricity in European countries. Second, I analyze different licensing schemes, namely pure ad-valorem royalties and pure fixed fees. I show that the two economic forces described in the previous sections are still there and regulate the strategic choice of the patent holder.

5.1 Price competition with capacity constraints

Assume firms are capacity constrained, and the constraint is a function of the technology employed. I analyze both the case where more efficient firms can ex-ante produce more units than less efficient rivals, and the opposite case.[15] Formally, each firm i can produce at most $k(c_i)$ units of the final good. Define ℓ as the last firm that can produce a positive output level at a marginal cost $c_\ell \in [\underline{c}, \bar{c}]$ before the market is completely served. The total quantity demanded in the market is fixed and equal to

$$Q = \int_{k(\underline{c})}^{k(c_{\ell-1})} k(c)\, dc + q(c_\ell)$$

where $q(c_\ell) \leq k(c_\ell)$ is the residual demand coverd by firm ℓ. All firms that are more efficient than ℓ produce at full capacity.[16] I further assume c_ℓ is strictly lower than $\bar{c}$. Taking the example of the day-ahead market for electricity, the demand must be covered, so that firms do not have the incentives to artificially create scarcity and raise the price.

Firms compete in prices to serve the market. As a consequence, the final price of the final good equals the marginal costs of the least efficient active company $P = c_\ell$. At this price, firms $c_i \in [\underline{c}, c_\ell)$ earn positive margins $c_\ell - c_i > 0$, whereas the *residual firm* c_ℓ makes zero profits.

As in the main model, an external innovator owns a patent for a technology that allows the production of the final good at $c^* < \underline{c}$, and licenses it with a pure per-unit royalty (r) contract. The new technology, if adopted by a firm i, would allow it to increase the margins earned per unit of the final good, without altering its capacity. For sake of brevity, I only focus on the non-exclusive

[15] This last case is common feature of energy sectors, where the capacity of renewable energy suppliers is generally lower than the capacity of standard fossil fuels suppliers, despite the much lower marginal costs of green energy.

[16] As firms have no mass, the expression $c_{\ell-1}$ in the integration upper limit should only be interpreted as the firm ranked immediately before c_ℓ.

dealing scenario.

I sort downstream firms into two separate groups: *active* and *inactive* firms. Let me generically denote the firms in the former group with technology c_i, and firms in the latter group with technology c_j. Active firms are those endowed with a technology $c_i \in [\underline{c}, c_\ell]$ that produce positive output, whereas inactive ones have technology $c_j \in (c_\ell, \bar{c}]$ and do not produce. The patent owner can offers a licensing contract to firms in both groups. Intuitively, licensing the technology to inactive companies is profitable if and only if they become active. Otherwise, selling zero units of final goods implies generating zero licensing revenues.

Active firms' maximum willingness to pay is simply given by $c_i - c^*$ and from each of them the patent owner earns $R_i = k(c_i)(c_i - c^*)$. One should notice that firm c_ℓ is only partially active, as it only produces $q(c_\ell)$ units to cover the residual demand. Instead, inactive companies earn zero profits ex-ante and, if they become active, they are willing to pay the cost difference between the residual firm's technology and the innovative one. Formally, their willingness to pay is always $c_\ell - c^*$. One should notice that, when an inactive firm becomes active, the residual firm $c_i = c_\ell$ stays active if and only if $k(c_j) < q(c_\ell)$, which is possible only if one assumes $k'_{c_i} < 0$, meaning that capacity constraint gets stricter the less efficient the firm. Otherwise, c_ℓ decreases to some $c_{\ell'} < c_\ell$ and available margins for active firms shrink.

The innovator has to decide the price r of the technology and (consequently) the number of contracts offered. If she decides to license only active firms, the problem resembles the one analyzed in section 3. Starting from the least efficient company c_ℓ, the innovator would identify a firm c_i such that the profit-maximizing number of contracts $m(c_i) = c_\ell - c_i$ and the price of technology is $r(c_i) = c_i - c^*$. All active firms less efficient than c_i wants to become licensee at price r_i. The problem of the innovator can be written as follows:

$$\begin{aligned} \max_{c_i} \; R^{ph} =& r(c_i)\, m(c_i) \left(q(c_\ell) + \int_{c_i}^{c_{\ell-1}} k(c) dc \right) = r(c_i)\, m(c_i)\, K(c_i) \\ \text{subject to} \quad & r(c_i) \leq c_i - c^* \end{aligned}$$

The first-order condition is very similar to equation 2. Moreover, one can easily see that choosing to offer one more royalty contract generates the notorious effects examined in this paper: first, it would increase the market penetration of the new technology, allowing the patent holder to collect more royalties (MSE); second, it would require to lower the price $r(c_i)$ of the technology (due to the firms' participation constraint), limiting the licensing revenues per contract (PE). Under some conditions on the support of costs distribution, c_i is interior and results in Proposition 1 hold.

If instead the innovator decides to serve only inactive companies, then the equilibrium price is always the one that equals the marginal costs of licensees with the residual active firm (all inactive firms would pay a price targeted to active ones). However, because new firms become active but the demand is given, the identity of the residual firm may change. More in particular, two different outcomes may occur. If $k'_{c_i} < 0$, then the number of active firms that would become inactive is lower than the number of inactive firms that would become active. Vice-versa, if $k'_{c_i} > 0$, the number of active firms that exit the market is larger than the number of new entrants. In both cases, the problem of the patent holder is to choose the number of licensees to maximize licensing revenues. Formally:

$$\begin{aligned}\max_{c_j} \ R^{ph} =& r(c_j) \left(\int_{c_{\ell+1}}^{\bar{c}-1} k(c)dc + q(\bar{c}) \right) = r(c_j)\, K(c_j) \\ \text{subject to} \quad & r(c_j) \leq c_\ell - c^*\end{aligned}$$

The problem of the patent holder is trivial only if the total capacity of inactive firms is less than the residual output produced by firm c_ℓ. In that case, all inactive firms enter the market, and the residual demand shrinks. The patent holder earns $K(c_j)(c_\ell - c^*)$.

Instead, when the aggregate capacity of inactive firms is larger than the residual demand, some active firms may have to leave the market if inactive firms start producing the final good. Indeed, fixed demand implies that the amount of output produced in the market remains unaltered. What changes is the composition of active firms producing the total output. Moreover, and more importantly, the identity of the residual firm changes as more inactive firms become active due to the new technology.

Define $c_{\ell''} < c_\ell$ as the residual active firm when all inactive companies enter the market and produce at $c^* + r(c_j)$. Then, we have the following proposition:

Proposition 3. *If $k'_{c_i} < 0$, there exist two firms $c_{\ell'} \in [c_{\ell''}, c_\ell]$ and $c_j^\dagger \in (c_\ell, \bar{c}]$ such that $c_\ell - c_{\ell'} < c_j^\dagger - c_\ell$ and $\int_{c_{\ell'}}^{c_\ell} k(c)\, dc = \int_{c_\ell}^{c_j^\dagger} k(c)\, dc$. The patent holder sets price $r = c_{\ell'} - c^*$ and signs $c_j^\dagger - c_\ell$ contracts.*

Proof. Because $k'_{c_i} < 0$, firms' efficiency positively determines their capacity. Hence, choose two firms that are equidistant from the residual, $c_\ell \pm x$. It must be that the total capacity $K_i^x = \int_{c_\ell - x}^{c_\ell} k(c)\, dc > K_j^x = \int_{c_\ell}^{c_\ell + x} k(c)\, dc$. When a mass of new capacity K_j^x becomes active, the same mass of capacity has to exit the market, as there is no additional demand. Because $K_j^x < K_i^x$, it must be that the number of firms becoming active is larger than the number of firms exiting the market. Define $x \equiv \bar{c} - c_\ell$ and $x_j^\dagger \equiv c_j^\ddagger - c_\ell$. We can identify two firms

$$c_{\ell''} = \{c_i \in (\underline{c}, c_\ell) \,||\, \int_{c_{\ell''}}^{c_\ell} k(c)\, dc = \int_{c_\ell}^{\bar{c}} k(c)\, dc\} \text{ and } c_{\ell'} = \{c_i \in (\underline{c}, c_\ell) \,||\, \int_{c_{\ell'}}^{c_\ell} k(c)\, dc = \int_{c_\ell}^{c_j^\dagger} k(c)\, dc\}$$

such that $c_{\ell'} > c_{\ell''}\ \forall\, c_j^\dagger < \bar{c}$.

The patent holder balances the usual trade-off between market penetration (MSE) and the size of the royalty rate (PE). Including more inactive firms in the licensing agreement implies the price of technology must fall, as more active firms leave the market and the residual demand is covered by a relatively more efficient firm. More in detail, if the price effect dominates the market share effect, the patent holder includes fewer inactive firms in the licensing agreement ($c_j^\dagger \downarrow$). Otherwise, if the MSE dominates the PE, the patent holder increases the number of contracts ($c_j^\dagger \uparrow$). The solution depends on the support of cost distribution and on the size of the demand. □

Consider now the case in which the capacity of a firm is lower than the capacity of less efficient rivals. Then, we derive the following proposition. Define $c_{\ell^{**}} < c_\ell$ as the residual active firm when all inactive companies enter the market and produce at $c^* + r(c_j)$. Then:

Proposition 4. *If $k'_{c_i} > 0$, there exist two firms $c_{\ell^*} \in [c_{\ell^{**}}, c_\ell]$ and $c_j^\ddagger \in (c_\ell, \bar{c}]$ such that $c_\ell - c_{\ell^*} > c_j^\ddagger - c_\ell$ and $\int_{c_{\ell^*}}^{c_\ell} k(c)\, dc = \int_{c_\ell}^{c_j^\ddagger} k(c)\, dc$. The patent holder sets price $r = c_{\ell^*} - c^*$ and signs $c_j^\ddagger - c_\ell$ contracts.*

Proof. Because $k'_{c_i} > 0$, firms' efficiency negatively determines their capacity. Hence, choose two firms that are equidistant from the residual, $c_\ell \pm x$. It must be that the total capacity $K_i^x = \int_{c_\ell - x}^{c_\ell} k(c)\,dc < K_j^x = \int_{c_\ell}^{c_\ell + x} k(c)\,dc$. When a mass of new capacity K_j^x becomes active, the same mass of capacity has to exit the market, as there is no additional demand. Because $K_j^x > K_i^x$, it must be that the number of firms becoming active is smaller than the number of firms exiting the market. Define $x \equiv \bar{c} - c_\ell$ and $x_j^\ddagger \equiv c_j^\ddagger - c_\ell$. We can identify two firms

$$c_{\ell^{**}} = \{c_i \in (\underline{c}, c_\ell) \,||\, \int_{c_{\ell^{**}}}^{c_\ell} k(c)\,dc = \int_{c_\ell}^{\bar{c}} k(c)\,dc\} \text{ and } c_{\ell^*} = \{c_i \in (\underline{c}, c_\ell) \,||\, \int_{c_{\ell^*}}^{c_\ell} k(c)\,dc = \int_{c_\ell}^{c_j^\ddagger} k(c)\,dc\}$$

such that $c_{\ell^*} > c_{\ell^{**}}\ \forall\, c_j^\ddagger < \bar{c}$.

The patent holder balances the usual trade-off between market penetration (MSE) and the size of the royalty rate (PE). Including more inactive firms in the licensing agreement implies the price of technology must fall, as more active firms leave the market and the residual demand is covered by a relatively more efficient firm. More in detail, if the price effect dominates the market share effect, the patent holder includes fewer inactive firms in the licensing agreement ($c_j^\ddagger \downarrow$). Otherwise, if the MSE dominates the PE, the patent holder increases the number of contracts ($c_j^\ddagger \uparrow$). The solution depends on the support of cost distribution and on the size of the demand. □

Condition $k'_{c_i} > 0$ implies that every inactive firm that becomes a licensee exerts severe negative pressure on the price of the final good and royalty rate. Because activating a new firm implies the shutdown of more than one firm, the PE is stronger, and we should expect lower diffusion of the technology than in the opposite case of $k'_{c_i} < 0$.

Finally, because the participation constraint of inactive firms is less strict than the one of active firms, the patent holder weakly prefers licensing inactive firms. Possibly, if all inactive firms are included in the licensing agreement, some active firms may start joining too. In this case, we are back to Proposition 1.

5.2 Ad-Valorem Royalty

Let us now focus on a different version of the royalty licensing scheme. Assume that now firms do not pay a price per unit of goods produced (and sold) with the new technology. Instead, they pay a share of the gross revenues - i.e., for every unit of the final good sold, the firms pay a share $r \in (0,1)$ of the price to the patent holder. In this case, the profit-maximizing patent holder needs to consider the size of r, the output produced, and the final good's price when choosing the targeted firm or group of firms. Assume that all the other assumptions of the model described in section 2 hold and, for simplicity, assume the patent holder deals exclusively with one firm. Then, her maximization problem can be written as:

$$\max_{c_i} \quad R^{ph} = r_i(c_i) P\left(Q^{tot}(r_i(c_i))\right) q_i(c^*, r_i(c_i))$$

where $Q^{tot}(r_i(c_i)) \equiv q_i(c^*, r_i(c_i)) + \int_{\underline{c}}^{\bar{c}} q(c)dc - q(c_i)$

Define $r_i \equiv r_i(c_i)$, $P \equiv P\left(Q^{tot}(r_i(c_i))\right)$, $Q^{tot} \equiv Q^{tot}(r_i(c_i))$, and $q_i \equiv q_i(c^*, r_i(c_i))$. Then, the maximization problem yields the following first-order condition:

$$\frac{d\,R^{ph}}{d\,c_i} = \underbrace{\left(\frac{d\,r_i}{d\,c_i} P + \frac{\partial\,P}{\partial\,Q^{tot}} \frac{\partial\,Q^{tot}}{\partial\,r_i} \frac{d\,r_i}{d\,c_i} r_i\right) q_i}_{\text{Price Effect}} + \underbrace{\left(\frac{\partial\,q_i}{\partial\,r_i} \frac{d\,r_i}{d\,c_i}\right) r_i P}_{\text{Market Share Effect}} = 0 \tag{4}$$

The first component on the left-hand side of equation (4) represents the PE, while the second component is the MSE. To determine the existence of a trade-off, one needs to understand the sign of the components of equation 4. First, it is important to assess the relationship between the royalty rate $r_i(c_i)$ and the ex-ante cost of production c_i. To do so, one needs to look at the participation constraint of the candidate adopters. Define Q^{dev} as the total output of firm i that decides to produce with the obsolete technology and not to pay for the license. The deviation is profitable if:

$$\Delta\,\pi_i = \underbrace{q(c^*)\,(P(Q(r_i)) - c^*) - r_i q(c^*) P(Q(r_i))}_{\text{profits with licensing}} - \underbrace{q(c_i)\left(P(Q^{dev}) - c_i\right)}_{\text{profits deviation}} \geq 0$$

$$\left(1 - \frac{c^*}{P(Q(r_i))}\right) - \frac{q(c_i)}{q(c^*)}\left(\frac{P(Q^{dev}) - c_i}{P(Q(r_i))}\right) \geq r_i$$

Hence, for any given level of royalty, the downstream firm accepts the offer if the above inequality is satisfied. The less efficient the licensee ex-ante ($c_i \uparrow$), the less strict the condition above, as $\partial q(c_i)/\partial c_i < 0$ and $\partial(P(Q) - c_i)/\partial c_i < 0$. In words, assume a patent holder sets a price of technology r_i which is accepted by firm i. The same price would always be acceptable for less efficient firms, but not always for more efficient ones. It is possible to can conclude that $\partial r_i/\partial c_i \geq 0$.

Given the relationship between the royalty rate and the ex-ante efficiency of the licensee, the MSE is non-positive. Instead, the analysis of the PE is a bit more complex. The first component is non-negative, as discussed above. The second component depends on the sign of $\partial Q^{tot}/\partial r_i$. In a linear demand model, the sign is negative and the PE is unambiguously positive.

5.2.1 Example: Duopoly with linear demand

The following cases are built upon the standard model of competition à la Cournot with linear demand function $P = A - q_i - q_j$. For clarity of exposition, mathematical steps are omitted and available in the appendix. As in section 3.1, assume two competing firms are labeled firm i and firm j. Without loss of generality, assume that firm i is endowed with better technology, such that its costs of production are $c_i < c_j$. An outsider patent holder sells a technology that enables firms to produce at a cost $c^* < c_i$. The firms compete in quantities for a homogeneous good. Their production levels coincide with the standard Cournot outcome:

$$q_i(c_i, c_j) = \frac{A - 2c_i + c_j}{3} \qquad\qquad q_j(c_j, c_i) = \frac{A - 2c_j + c_i}{3}$$

The price of the technology is a share of the revenues per unit of the final good sold - i.e., the licensee k pays r times $P(Q)\,q_k$.

Firm i gets the license. Assume the patent holder licenses the new technology to the most efficient firm, namely, firm i. Then, the patent holder's profits are:

$$R_i^{ph} = \frac{r_i^*((A+c_j)(1-r_i^*)+c^*)((A+c_j)(1-r_i^*)-2c^*)}{9(1-r_i^*)^2}$$

Firm j gets the license. Assume the patent holder licenses the new technology to the least efficient firm, namely, firm j. Then, the patent holder's profits are:

$$R_j^{ph} = \frac{r_j^*((A+c_i)(1-r_j^*)+c^*)((A+c_i)(1-r_j^*)-2c^*)}{9(1-r_j^*)^2}$$

Both firms i and j get the license. Assume now that the patent holder is not capable of dealing with one firm exclusively. Thus, when he charges a price r which is compatible with the Participation constraint of both firms i and j, they will adopt the new technology simultaneously. In this case, the patent holder's profits are:

$$R_{ij}^{ph} = \frac{2r_{ij}^*(A+2c-r_{ij}^*)(A-c-r_{ij}^*)}{9(1-r)^2}$$

Comparison of the results Let us first assume that the optimal fee r satisfies the P.C. of both firms in all the analyzed scenarios. It is apparent that, in this case, the patent holder prefers licensing the innovation to all firms or, in the case of exclusive dealing, to the most efficient one. In fact, if one zeroes out the P.E. by preventing the patent holder from exploiting differences in the firm's willingness to pay, then the only strategy left is to maximize the MSE by signing licensing contracts with all the participants in the downstream market. However, once I account for asymmetries in the willingness to pay, the game's outcome changes. In other words, by analyzing the parametric regions where the patent holder is constrained by the P.C. of the downstream firms, it is possible to derive conditions for which the equilibrium is to license the least efficient firm instead of the most efficient one or both.

Let us analyze the following numerical examples. First, I normalize the intercept of the demand function $A = 1$ and consider the following values of firms' costs $c^* = 0.1$, $c_i = 0.18$, and $c_j = \mathbf{0.22}$.

Then, royalties are $r_i^* = 0.22$, $r_j^* = 0.309$, and $r_{ij}^* = 0.22$. In this case, the equilibrium of the game is to sell the technology to all firms, as $R_{ij}^{ph} = 0.054 > 0.041 = R_j^{ph}$ ($R_i^{ph} = 0.03$).

Second, I consider the same parameters value, except for $c_j = \mathbf{0.32}$. Then, royalties are $r_i^* = 0.196$, $r_j^* = 0.501$, and $r_{ij}^* = 0.22$. In this case, the equilibrium of the game is to sell the technology only to the least efficient firm, as $R_{ij}^{ph} = 0.054 < 0.065 = R_j^{ph}$ ($R_i^{ph} = 0.033$).

This result suggests that, in line with Lemma 1, also when the patent holder licenses her technology via ad-valorem royalties, the asymmetry in ex-ante firms' efficiency generates a PE that can be crucial to determine the outcome of the licensing game. In particular, the greater the ex-ante asymmetry, and the greater the differential between the new technology and the best one available on the market, the stronger the price effect. As a consequence, it is more likely that an inefficient firm becomes a licensee.

5.3 Fixed fee

Fixed fee licensing represents a far less interesting case. The patent holder sets a fee that is equal to the difference between the achievable payoff with the new technology and the ex-ante payoff with the endowed technology. The payoff of a firm increases in the efficiency gap between itself and the less efficient rival. As a consequence, the larger the number of adopters, the lower the willingness to pay for the new technology. This feature of fixed fee contracts implies that even with homogeneous firms, the trade-off between PE and MSE drives the patent holder's decision about the diffusion of the new technology.[17] Formally, the payoffs of the firms without and with the new technology are

$$\pi(c_i) = (P(Q) - c_i)\, q(c_i); \qquad \pi(c^*) = (P(Q^*(m)) - c^*)\, q(c^*) - F$$

Where $Q^*(m)$ indicates the industry output when $n \geq m \geq 1$ firms adopt the new technology with The participation constraint t is:

$$F \leq \pi(c^*) - \pi(c_i) = (P(Q^*(m)) - c^*)\, q(c^*) - (P(Q) - c_i)\, q(c_i)$$

[17] See Sandrini (2022) for a formal analysis of the trade-off with fixed fee licensing contracts and homogeneous firms.

Assume firms are homogeneous, meaning that $c_i = c_{-i} = c$. Then, $(P(Q) - c)\, q(c)$ is common to all firms. In this case, what alters the participation constraint t is the effect of adoption on the market price. In particular, when many firms adopt the new technology the industry output expands and the market price falls. As a consequence, the markup that a firm can obtain with the new technology $(P(Q^*(m)) - c^*)$ declines with the number of adopters m. The patent holder has to balance the trade-off between collecting more small fees (MSE) and collecting fewer large fees (PE). The trade-off is more evident when firms are heterogeneous.

6 Conclusion

This article challenges the standard result that royalty licensing of a process innovation leads to the full diffusion of superior technology. Moreover, I disentangle two main forces: a price effect and a market share effect. The former identifies the inefficient firms' higher willingness to pay for superior technology, as their cost reduction effect would be more significant than for efficient firms. Instead, the market share effect suggests that providing the most efficient firms with the innovation allows the patent holder to maximize the volume of royalties collected, as efficient licensees have a larger market share and sell more units of the final good.

The dominance of these two forces determines which firms (and how many) gets the license in equilibrium. Interestingly, this article shows that full diffusion of innovation with royalty licensing is a particular result that depends on how close firms' embedded technologies originally are. If one considers firms that are very heterogeneous, the patent holder may have a strong incentive to increase the price of the technology and to serve fewer firms. Similarly, if the novel technology is not excessively more productive than the best one available in the market, the price effect tends to dominate the market share effect and not all companies become licensees.

From a welfare perspective, licensing increases consumer surplus. By lowering the price of the technology (royalty rate), the innovator limits her or his ability to extract the rent generated by the innovation, which is partially captured by firms. Consequently, because firms are on average more efficient, the price of the final good falls.

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Mathematical appendix

6.1 Ad-valorem Royalty - Duopoly with linear demand function

Firm i gets the license. Assume the patent holder licenses the new technology to the most efficient firm, namely, firm i. Then, the corresponding output levels become:

$$q_i(c^*, r, c_j) = \frac{(A + c_j)(1 - r) - 2c^*}{3(1 - r)} \qquad q_j(c_j, c^*, r) = \frac{(A - 2c_j)(1 - r) + c^*}{3(1 - r)}$$

The associated level of profits of the downstream firms can be written as

$$\pi_i = \frac{((A + c_j)(1 - r) - 2c^*)^2}{9(1 - r)} \quad \pi_j = \frac{((A - 2c_j)(1 - r) + c^*)^2}{9(1 - r)^2}$$

The maximum r the patent holder can charge must satisfy the Participation constraint t of the licensee. In practice,

$$r \leq r_i^* \quad s.t. \quad \frac{((A + c_j)(1 - r^*) - 2c^*)^2}{9(1 - r^*)} \geq \frac{(A - 2c_i + c_j)^2}{9}$$

where $r_i^* \equiv -\frac{4(A+c_j)(c-c_i)+(A-2c_i+c_j)\sqrt{8c(A+c_j)+(A-2c_i+c_j)^2}+4c_i^2-(A+c_j)^2}{2(A+c_j)^2}$. The patent holder's profits are:

$$R_i^{ph} = \frac{r_i^*((A + c_j)(1 - r_i^*) + c^*)((A + c_j)(1 - r_i^*) - 2c^*)}{9(1 - r_i^*)^2}$$

Firm j gets the license. Assume the patent holder licenses the new technology to the least efficient firm, namely, firm j. Then, the corresponding output levels become:

$$q_i(c_i, c^*, r) = \frac{(A - 2c_i)(1 - r) + c^*}{3(1 - r)} \qquad q_j(c^*, r, c_i) = \frac{(A + c_i)(1 - r) - 2c^*}{3(1 - r)}$$

The associated level of profits of the downstream firms can be written as

$$\pi_i = \frac{((A - 2c_i)(1 - r) + c^*)^2}{9(1 - r)^2} \quad \pi_j = \frac{((A + c_i)(1 - r) - 2c^*)^2}{9(1 - r)}$$

The maximum r the patent holder can charge must satisfy the Participation constraint t of the licensee. In practice,

$$r \leq r_j^* \quad s.t. \quad \frac{((A+c_i)(1-r^*)-2c^*)^2}{9(1-r^*)} \geq \frac{(A-2c_j+c_i)^2}{9}$$

where $r_j^* \equiv -\frac{4(A+c_i)(c-c_j)+(A-2c_j+c_i)\sqrt{8c(A+c_i)+(A-2c_j+c_i)^2}+4c_j^2-(A+c_i)^2}{2(A+c_i)^2}$. The patent holder's profits are:

$$R_j^{ph} = \frac{r_j^*((A+c_i)(1-r_j^*)+c^*)((A+c_i)(1-r_j^*)-2c^*)}{9(1-r_j^*)^2}$$

Both firms i and j get the license. Assume now that the patent holder is not capable of dealing with one firm exclusively. Thus, when he charges a price r which is compatible with the Participation constraint t of both firms i and j, they will adopt the new technology simultaneously. In this case, the corresponding output levels become:

$$q_i(c^*, r) = \frac{A-c-r}{3(1-r)} \qquad q_j(c^*, r) = \frac{A-c-r}{3(1-r)}$$

The associated level of profits of the downstream firms can be written as

$$\pi_i = \frac{(A-c-r)^2}{9(1-r)} \quad \pi_j = \frac{(A-c-r)^2}{9(1-r)}$$

The maximum r the patent holder can charge must satisfy the Participation constraint t of the licensee. In practice,

$$r \leq r_{ij}^* \quad s.t. \quad \frac{(A-c-r)^2}{9(1-r)} \geq \frac{((A-2c_i)(1-r)+c^*)^2}{9(1-r)^2}$$

where $r_{ij}^* \in [r_i^*, r_j^*]$. The patent holder's profits are:

$$R_{ij}^{ph} = \frac{2r_{ij}^*(A+2c-r_{ij}^*)(A-c-r_{ij}^*)}{9(1-r)^2}$$